Spatial networks with random connections

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Many networks of current interest have a spatial structure, in that the nodes and/or links are located in physical space. Examples include climate, communications, infrastructure, nanowire, neuronal and transport networks. An early and still popular model of spatial networks is the random geometric graph, where nodes are located randomly and links formed between sufficiently close nodes.
Recent studies have considered random connection models, in which there is a link probability $H(r)$, a function of the mutual node distance r. The links are chosen independently and form a second source of randomness (in addition to the node locations). In principle, a distance-dependent link probability can be defined in any spatial network. Also, the (non-spatial) constant function $H(r) = p$ leads to the well-studied Erdos-Renyi model.
One important application is to wireless ad-hoc networks, in which devices (nodes) communicate directly with each other rather than with a central router; this can enhance scalability, flexibility and power requirements. These have been used in for example sensor networks, vehicular networks and robot swarms. There are detailed models based on physical propagation assumptions; one of the simplest is Rayleigh fading, which leads to

$$H(r) = e^{-(r/r0)^{\eta}}$$

where $2 \leq \eta \leq 6$ is a constant called the path loss exponent. The limit eta to infinity leads back to the original random geometric graph.
Network characteristics can now be investigated as a function of the model parameters. The overall connection probability is well approximated by considering the expected number of isolated nodes. Applying Laplace's method to relevant multidimensional integrals, we find that it can be estimated from just a few moments of $H(r)$ for a wide variety of domain geometries. Furthermore, there are qualitative differences as a result of the random connections. In particular, the more realistic model allows a more accurate estimation of connectivity and resilience (as quantified by k-connectivity) than the original.
Recent work has also considered networks where the nodes are mobile and/or inhomogeneously distributed. We investigate when this breaks the connection between connectivity and isolation, and find it leads to a number of interesting and largely unexplored phenomena.

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