On the statistical model of the atom

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The Thomas-Fermi statistical model for the ground state of an atom has been usually used for the determination of its non-uniform electron density n(r) and self-consistent electric field V(r). This model assumes that the functions n(r) and V(r) vary slowly enough within an electron De Broglie wavelength, so that the quasi-classical description can be used. The Thomas-Fermi model has proved to be very useful in deriving properties such as the binding energy of heavy atoms. Besides, after suitable modifications, it has been applied to molecules, solids and nuclei. One of the most remarkable difficulties to overcome in order to get results is due to the determination of the self-consistent space-dependent field deriving by the interaction among the electrons of the atom. This field, that has to satisfy the Poisson Equation, has been previously solved only numerically, and then it has been used to calculate the total binding energy of the atoms electron cloud. In this work we propose a different method that, starting from the Fermi-Dirac distribution function of the completely degenerate state of the electron cloud, allows to obtain the space-dependent electron density function n(r). This electron density, which contains the chemical potential too, is obtained straight from the model assumptions without any lack of strictness, and overlaps exactly to what has been obtained in previous works. Then, with the purpose of gaining deeper insight into the physics of the problem, we introduce an approximation for the electron density, and thereafter we solve analytically the Poisson Equation. This analytical solution of the homogeneous part of the Poisson equation, yielding the self-consistent electric field V(r), turns out to be a Bessel function. Besides, we of course calculated the particular integrals for satisfying also the boundary conditions imposed by the physics of the problem. Now, an important and nontrivial point fulfilled by our model is the condition represented by the finite number of the electrons. Hence a finite value for the radius of the electron cloud comes naturally out. Now, from all this, upon integrating over the volume the electron density, also the discrete energy levels of the completely degenerate cloud of electrons in the extra-nuclear space and the ionization potentials of the atom are obtained. This model can also be applied to a completely degenerate plasma and obtaining for instance the pair correlation function for the electrons.

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