Nonequilibrium properties of Levy noises

E. Gudowska-Nowak
Marian Smoluchowski Institute of Physics, Jagiellonian University in Krakow, Poland.

A system at equilibrium exhibits microscopic reversibility, i.e. any path in phase space is just as often traversed in one direction as that it is traversed in the backward time direction. On the other hand, microscopic reversibility implies a more general condition of the detailed balance which is the requirement that net flow between any two states is zero. Following definitions of detailed balance and microscopic reversibility, we show how Gaussian white noise can be associated with microscopic reversibility and how the presence of more general Levy fluctuations can lead to its violation. In our example the Levy noise itself is symmetric in the sense that each fluctuation in the positive direction is as likely as its negative-direction counterpart. However, when applied to a dynamic system within the Langevin formalism, the non-Gaussian Levy white noise leads to breaking of the time reversal symmetry pointing to its inherent nonequilibrium character. We extend this analysis to investigations of fluctuations in mechanical energy of a linear oscillator perturbed by Levy noise. Derived distributions of kinetic and potential energies in stationary states clearly deviate in this case from equipartition showing nontrivial correlation between the positions and velocities. We further discuss thermodynamic interpretation of energy flows in the system. In particular, we demonstrate that with Levy fluctuations the concepts of thermalization and equilibrium need be addressed with much care. First, Langevin equations driven by non-Gaussian Levy white noises lead to non-Gibbsian stationary distributions. Next, the Levy noise has infinite variance and so there is no standard fluctuation-dissipation relation to connect the variance of the noise to the strength of the friction. This pathology does not take place however in cases when the noise is regarded as an external pulse-like forcing and not just as a part of the heat bath for the system at hand. Altogether our work shows that actions of Levy fluctuations drive systems away from equilibrium states and are strongly manifested even in weakly perturbed linear systems.