Zero-temperature directed polymer in random potential on higher dimension

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Zero-temperature directed polymer in random potentials on $d = 4 + 1$ dimension is described. Consider a discrete directed polymer model on a discrete "hyper-pyramid" structure with random potential $\mu(\mathbf{x}, t)$ assigned to each site $(\mathbf{x}, t)$ where $\mathbf{x}$ is the $d - 1$ dimensional transverse vector and t is the longitudinal length of the polymer.

The polymer starts from the substrate at $t = 0$ and its path is restricted by $|\mathbf{x}(t) - \mathbf{x}(t + 1)| = 0$ or 1. There is a bending energy γ against a transverse jump $|\mathbf{x}(t) - \mathbf{x}(t+1)| = 1$. It represents the stretched energy of the polymer for the transverse jump. As an initial condition, $E(\mathbf{x}, t = 0) = 0$ is given. Then the minimum energy $E(\mathbf{x}, t)$ among all polymers arriving at $(\mathbf{x}, t)$ can be obtained recursively.

Here, we have presented numerical analysis of the directed polymers in $4 + 1$ dimensions. The energy fluctuation $\Delta E(t)$ of the polymer grows as t^β as function of polymer length t with $\beta = 0.158 \pm 0.007$ and ΔE follows $\Delta E(L) \sim L^\alpha$ at saturation with $\alpha = 0.272 \pm 0.009$, where L is the system size. The dynamic exponent $z = \alpha/\beta \approx 1.72$ is obtained. The estimated values of exponents satisfy the scaling relation $\alpha + z = 2$ very well. Our results show that the upper critical dimension of the Kardar-Parisi-Zhang Equation is higher than $d = 4 + 1$ dimension.

It is known that the directed polymer problem in random potentials belongs to the KPZ universal class. Our results are good agreement with the recent results $\beta \approx 0.158$, $\alpha \approx 0.273$ from the RSOS model in $4+1$ dimensions. The estimated β is slightly less than but close to our conjecture 1/6. Considering the finite size effects, our numerical data do not exclude the conjecture. We also estimate z independently from the end to end fluctuation of the path using $\Delta X \sim t^{2/z}$ and obtain $z \approx 1.73$. The transverse fluctuation of the polymer becomes super diffusive.

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