Subdiffusion-reaction process in a composite system with a thin membrane

T. Kosztolowicz[1], K.D. Lewandowska[2]

[1]Institute of Physics, Jan Kochanowski University in Kielce, Poland

[2]Department of Radiological Informatics and Statistics, Medical University of Gdansk, Poland

We consider subdiffusion–reaction process in a composite system consisting of two homogeneous media which are separated by a partially permeable thin membrane. Subdiffusion parameters and reaction rates can be different in both media. In each part of the system the process under consideration is described by means of the subdiffusion–reaction equations with a fractional Riemann–Liouville time derivative. In order to derive the Greens' functions for the considered process we use a simple random walk model with reactions in a system with discrete both time and space variables. Particles' random walk is then described by a set of difference equations which can be solved by means of the generating function method. We assume that a particle performs its single jump at a discrete time to the least neighboring sites only. Additionally, a particle is not allowed to stay at any discrete position when the time of a jump is achieved with the exception of stopping a particle by a membrane what takes place with a some probability; then the particle remains in its position. Using the generating function obtained for the discrete equations mentioned above we pass from discrete to continuous time and space variables by means of the procedure presented in this contribution. Based on the obtained functions we derive boundary conditions at the border between media. One of them demands a continuity of a flux and the other takes rather unexpected form which contains fractional time derivatives.

We also show that a discrete model of random walk appears to be a very useful tool in modeling subdiffusion or normal diffusion. A definite advantage of this method is as follows. We can obtain the Greens' functions without necessity of assuming boundary conditions which are required when solving subdiffusion–reaction equations. The other advantage of this method is that processes described by it have a relatively simple interpretation. The presented method can be also useful in describing the various subdiffusion–reaction processes occurring in biology. We use this model to describe the process of releasing drugs from a subdiffuion medium into a medium in which normal diffusion occurs through a thin membrane.

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