Extension of black hole thermodynamics with information geometry

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Information geometry represents probabilities with Riemannian metric spaces. This is an idea dating back to the work of Fisher and Rao. That the resulting stochastic manifolds have applications in statistical mechanics and thermodynamics, with their probabilistic structures, is thus no surprise. My talk involves the logical theme that the unique Ricci curvature scalar $R$ of these stochastic manifolds extends the reach of thermodynamics. I propose that it illuminates fluctuations at mesoscopic size scales. I put special emphasis on some recent ideas in black hole thermodynamics, a field of study originated by Bekenstein, Hawking, and others. Despite much effort by the scientific community, there is no consensus on any underlying microstructure at the foundation of black hole thermodynamics. This constitutes a significant gap in physical theory. Thermodynamic fluctuation theory, extended with ideas from information geometry, offers perhaps some insight for this physical problem.

Thermodynamic Ricci curvature $R$ is an element of thermodynamic metric geometry, originated by Weinhold in 1975. Weinhold introduced a thermodynamic energy inner product. Ruppeiner then wrote a Riemannian thermodynamic entropy metric to represent thermodynamic fluctuation theory, a branch of information geometry, and began systematically calculating the thermodynamic Ricci curvature scalar $R$. A parallel effort was by Andresen et al. who began the systematic application of the thermodynamic entropy metric to characterize finite-time thermodynamic processes.

There is substantial evidence that $R$ is a thermodynamic measure of intermolecular interactions. It appears that $|R|$ gives the characteristic size of organized mesoscopic fluctuations in fluid and magnetic systems. The sign of $R$ also appears to be significant: for fluid systems, $R$ is positive/negative for systems in states dominated by repulsive/attractive intermolecular interactions. $R$ for systems with no interactions between microscopic constituents have $R = 0$.

Information geometry, via the invariant thermodynamic curvature $R$, presents a unifying theme encompassing fluid systems, magnetic systems, and perhaps black holes.