Diffusion on surfaces: from theory to experiments and back

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Diffusion is an ubiquitous phenomena. It permeates physics, mathematics chemistry, biology and even sociology and economics. In physics, it has been associated to several works made by scientists like Maxwell, Stefan, Einstein, Smoluchowski and Boltzmann, among others. Over the past decade, great theoretical attention has been given to the possibility, as in general relativity, that the curvature of the environment could change a particle diffusing behavior \cite{1,2}. For instance, It was found that the root mean square displacement, which behaves linearly with time for the classical Euclidean diffusion phenomena, may have polynomial corrections if the diffusion occurs in a curved surface, e.g. for a sphere, as demonstrated in \cite{1}. Very recently, these theoretical predictions were verified experimentally using single-particle tracking for a nanoparticle diffusing on a spherically curved oil/water interface by Zhong et al.\cite{3}. However, for some specific radii of the spherical water-oil interface, an unexpected behavior was observed for the root mean square displacement that requires further theoretical investigation. In fact, It was observed that that the diffusion slows down significantly when the oil droplet becomes smaller \cite{3}. It is our aim in this work trying to theoretically explain this novel experimental finding.

Since the nanoparticle size can be commensurable with the spherical oil droplet size, we need to take into account the perturbations on the interface metric due to the spherical nano-particle volume. We symbolically computed the corrections to the root mean square displacement in the perturbed metric, using a perturbed metric analogous to the one in \cite{2}. By doing so, we try to explain analytically the recent unexpected experimental results found by Zhong \textit{et al} in \cite{3}. This phenomena may have potential applications in the context of biological, interface and surface physics.

\begin{thebibliography}{99}
  \bibitem{1} P. Castro-Villarreal, JSTAT \textbf{08}, 006 (2010).
  \bibitem{2} N. Ogawa, Phys, Rev. E \textbf{81}, 061113 (2010).
\end{thebibliography}