

## Energy scaling in optimal control of networks

I. Klickstein, A. Shirin, F. Sorrentino

University of New Mexico, US

Recently, it has been shown that the control energy required to control a large dynamical complex network is prohibitively large when there are only a few control inputs. Most methods to reduce the control energy have focused on where, in the network, to place additional control inputs. We show that by controlling the states of a subset of the nodes of a network, rather than the state of every node, the required energy to control a portion of the network can be reduced substantially. The energy requirements exponentially decay with the number of target nodes, suggesting that large networks can be controlled by a relatively small number of inputs as long as the target set is appropriately sized. We also see that the control energy can be reduced even more if we relax the constraint that the prescribed final states are satisfied strictly. We introduce a new control strategy called balanced control for which we set our objective function as a convex combination of two competitive terms: (i) the distance between the output final states at a given final time and given prescribed states and (ii) the total control energy expenditure over the given time period. We also see that the required energy for the optimal balanced control problem approximates the required energy for the optimal target control problem when  $\alpha$ , the coefficient of the second term, is very small. We discuss the effect of several parameters on the minimum balanced control energy, and in particular the time horizon and the number of input nodes. We extend our study to real datasets. We see that for small values of  $\alpha$ , in the case of metabolic networks, the control energy for balanced control reduces significantly from the output control energy. On the other hand, the Food Webs and Social networks are not benefited as much as the balanced control energy remains approximately the same, in comparison. However, for large values of the parameter  $\alpha$ , all of the networks need approximately the same amount of energy for balanced control. We validate our conclusions in model and real networks regardless of system size, energy restrictions, state restrictions, input node choices, and target node choices.

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[2] Klickstein, Shirin, Sorrentino, *Nature Communications*, to appear (2017).

[3] Liu, Barabasi, *Rev. Mod. Phys.* **88**, 035006 (2016).