Role of heterogeneity in stochastic agent-based models

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Most applications of Statistical Mechanics methods to agent-based models make assumptions that aim at a simplification of the mathematical treatment and which are reasonable, or well established, in other applications of the field. Amongst others, we can cite the assumption of the thermodynamic limit and the assumption that there is a high degree of homogeneity amongst the agents. This is certainly not true in most cases: the number of agents is never close to the Avogadro number and the dispersion in the individual features of agents is an unavoidable nature of the system. In this talk I will discuss some difficulties associated to the existence of such a heterogeneity and the mathematical tools that can be used to achieve analytical results. As an example, I will consider in detail both network and parametric heterogeneity in Kirmans model for herding behavior in financial markets. Stylized facts of financial markets (fat tails, volatility clustering) has been proposed as an emergent phenomenon of interactions among traders. One of the simplest agent-based models capable of reproducing these statistical properties is the one proposed by Kirman. The fundamental aspect of the model is that agents change opinion based on the proportion of neighbor agents holding it. The effect of network structure on the results of the model is also addressed with recent analytical tools known as heterogeneous mean field approximations. This approach suggests that the dynamics in an heterogeneous degree network is equivalent to the usual all-to-all approximation with an effective system size $N_{\text{eff}} = N\mu_k^2/\mu_2$, where $\mu_k$ is the k-th moment of the degree distribution. This implies that highly heterogeneous degree networks are characterized by a low effective population number. Intuitively, only highly connected agents play an important role in the dynamics and the number of those agents is measured by this effective population number. Taking into account that most real networks are highly heterogeneous with power-law degree distributions, one concludes that the effect finite-size fluctuations is non-trivial and must be studied in detail for each specific type of network.