Does the spin-glass transition survive in a magnetic field? Numerical results in five and six dimensions

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The existence and nature of a spin-glass phase transition in the presence of an external magnetic field remains one of the longstanding unresolved problems in the statistical physics of disordered systems. While mean-field theory predicts the persistence of a transition along the de Almeida–Thouless line, its fate in finite dimensions is still under active debate, with competing theoretical scenarios and, at times, contradictory numerical results.

In this talk, I will present recent large-scale Monte Carlo simulations of the Edwards–Anderson Ising spin glass in five and six spatial dimensions. In both cases, our results provide evidence compatible with a second-order phase transition in the presence of a magnetic field. In six dimensions—traditionally identified as the upper critical dimension by classical field-theoretical arguments, although recent analyses have suggested $D_U = 8$—our data support critical scaling and allow for estimates of the associated critical exponents. In five dimensions, we also find numerical indications of a transition, both in the presence and in the absence of a field. Taken together, these results place nontrivial constraints on the possible scenarios for the phase diagram in a magnetic field.

A central aspect of our analysis is the comparison with predictions from replica field theory. In particular, we study the behavior of the ratio of renormalized cubic couplings, which provides a sensitive probe of the nature of the transition. Our results are consistent with the expectations for a continuous transition. We also find qualitative differences in scaling behavior with respect to the zero-field case, as well as between five and six dimensions, most notably the anomalous role of zero-momentum modes, which induce strong corrections to finite-size scaling and complicate the interpretation of standard observables.

Beyond presenting these results, I will review the broader theoretical and numerical landscape of the problem. This includes renormalization-group analyses of replica-symmetric effective field theories and their logical interrelations, as well as alternative approaches such as the M-layer expansion around the Bethe lattice. From the numerical perspective, I will discuss both classical and recent results for finite-dimensional Edwards–Anderson models, as well as studies of one-dimensional spin-glass models with power-law interactions, which are often used as proxies for finite-dimensional short-range systems.

Overall, our findings point toward a coherent—yet still incomplete—picture of spin-glass criticality in a magnetic field. I will conclude by outlining the main open questions and possible future directions.

References:

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[2] M. Aguilar-Janita, V. Martín-Mayor, J. Moreno-Gordo, and J. J. Ruiz-Lorenzo, Evidence of a de Almeida–Thouless line below six dimensions, J. Stat. Mech. 2025, 113301 (2025).