# Laplace Nework Renormalization in functional brain Networks

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Understanding the macroscopic organization of the brain from its microscopic and mesoscopic dynamics remains one of the greatest challenges in neuroscience. The Renormalization Group (RG) [1], as a tool for describing critical behavior, offers a rigorous framework to investigate how system properties emerge through successive scale changes. The application of this technique to neural systems is motivated by evidence that the brain operates near a critical point, optimizing information processing. Recently, fundamental studies have established important milestones by applying phenomenological renormalization procedures based directly on neuronal activity and fMRI time series, identifying scale invariance across different renormalization stages [2,3].

Despite these advances, time-series-based renormalization cannot be directly applied to brain networks, and therefore may not capture the topological complexity of brain networks. The present work adress this limitation by specifically focusing on direct renormalization of the brain network, utilizing the recently developed Laplacian Renormalization Group (LRG) [4]. Unlike phenomenological approaches, LRG employs the spectrum of the Laplacian operator to define resolution scales based on diffusion times. This allows for coarse-graining to be performed directly on the connectivity structure.

This investigation, currently under development, utilizes high-resolution human fMRI data from Schaefer atlas and 1000 cortical regions [5] to reconstruct functional graphs and subject them to the LRG flow. The central objective is to observe how the network architecture transforms during the renormalization of slow modes and whether it preserves invariant topological properties that time-series analyses alone might omit.

References:

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