Hamiltonian Systems of Many Degrees of Freedom: From Classical to Statistical Mechanics

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Understanding how macroscopic statistical behaviour emerges from microscopic deterministic dynamics remains a central problem in the statistical mechanics of complex systems. In this contribution, we examine this question in the context of nonlinear Hamiltonian systems with many degrees of freedom, focusing on the mechanisms that govern the transition from regular motion to chaos and their implications for ergodicity and thermodynamic behaviour.

We begin with low-dimensional paradigms, such as the Hénon-Heiles system, to illustrate the coexistence of ordered and chaotic trajectories and the organising role of simple periodic orbits (SPOs) in phase space. Extending to high-dimensional systems, we analyse prototypical nonlinear lattices, including the Fermi-Pasta-Ulam (FPU) and discretised Gross-Pitaevskii (BEC) Hamiltonians. Using analytical approaches based on symmetry considerations and linear stability analysis, we identify families of SPOs and determine their destabilisation thresholds, which scale with system size and energy density, signalling the onset of weak chaos.

As the system evolves towards higher energy densities, we demonstrate the emergence of strong chaos through the interaction and eventual merging of chaotic regions associated with different SPOs, reflected in the convergence of their Lyapunov spectra. In this regime, we show that the Kolmogorov-Sinai entropy scales linearly with the number of degrees of freedom, establishing its extensivity and supporting the emergence of ergodic behaviour in the thermodynamic limit.

To characterise the dynamics efficiently, we employ the Smaller Alignment Index (SALI), which provides a robust and computationally efficient criterion for distinguishing between regular and chaotic motion in high-dimensional phase spaces.

Overall, the results provide a coherent dynamical framework linking local instability mechanisms to global statistical behaviour, offering insight into the foundations of statistical mechanics in nonlinear many-body systems.