

First return processes on random networks

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First return (FR) processes in which a stochastic walker returns to its starting point for the first time are fundamental for the understanding of diffusion and search dynamics. They are the building blocks of all return dynamics - via renewal theory they generate the full return statistics, and their distribution governs the mixing times of the system. On regular geometries, such as the one-dimensional lattice, combinatorial methods yield closed-form expressions for the return statistics, reflecting the underlying recurrence or transience. Extending these studies to disordered landscapes such as those encountered in random networks, the distribution of first return times reveals deep connections between structural features and dynamical behavior. Their relevance to search processes stems from the fact that first return events determine both the efficiency of exploration and the likelihood of redundant visits, thus providing key insights into how quickly and thoroughly a walker can cover a given structure. In this talk I will present analytical results for the distribution of first return (FR) times of random walks (RWs) and on random networks. I will show that classical RWs exhibit two types of FR trajectories: retroceding trajectories in which the RW retrocedes its own steps backwards to the initial point and non-retroceding trajectories in which it returns via at least one cycle [1]. While tree networks (such as the Bethe lattice) accommodate only retroceding trajectories, general random networks such as the configuration model accommodate both. To reveal the distinction between the two types of trajectories, I will present analytical results for the distribution of FR times of non-backtracking walks (NBW's), which exhibit only non-retroceding trajectories [2]. These results provide useful insight on the advantages of NBWs which scan the network faster and more efficiently than classical RWs. As a result, they are exceptionally useful for network exploration, sampling procedures and search processes on networks.

References:

- [1] I. Tishby, O. Biham, E. Katzav, Analytical results for the distribution of first return times of random walks on random regular graphs, *J. Phys. A* 54 325001 (2021).
- [2] D. Lev-Ari, I. Tishby, O. Biham, E. Katzav and D Krapf, Analytical results for the distribution of first return times of non-backtracking random walks on configuration model networks, *J. Phys. A* 58 505002 (2025).