

The Hamiltonian mechanics of exotic particles

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We develop a Hamiltonian formulation of classical mechanics on Aristotelian manifolds [1] - geometric backgrounds that possess absolute notions of time and space and thus lack local boost symmetry. As a consequence, net spatial momentum becomes an intrinsic, frame-independent variable that cannot be eliminated by a change of coordinates. Such structures arise naturally in systems where a preferred rest frame is physically distinguished. Our work provides a systematic phase space construction adapted to these backgrounds and establishes the foundations of Hamiltonian dynamics and kinetic theory in this broader setting.

Symmetries and conservation laws are analyzed geometrically. We demonstrate that conserved quantities arise from lifted Killing vectors associated with symmetries of the underlying manifold. This provides a clear and systematic correspondence between spacetime symmetries and conserved phase space charges, extending familiar results from relativistic and Galilean settings to the Aristotelian case.

We then extend the formalism to kinetic theory. Starting from the invariant phase space measure, we construct distribution functions and derive macroscopic currents. At leading order, we show that the charge current and stress tensor reproduce the equations of boost-agnostic ideal hydrodynamics [2, 3]. Remarkably, the ideal gas law also emerges universally, independent of microscopic details, reflecting the geometric structure of the underlying dynamics rather than boost symmetry.

Our framework thus supplies a unified geometric and dynamical foundation for systems in which boost invariance is absent. This setting is particularly natural for active matter [4], where constituents continuously consume energy and generate motion relative to a preferred laboratory frame. In such systems, self-propulsion singles out a physical rest frame and net momentum cannot, in general, be eliminated by a change of coordinates.

Beyond pure physics, the formalism also suggests a geometric generalisation of particle swarm optimization [5, 6]. In this setting, the free "gas-like" component of swarm motion arises from an invariant Hamiltonian flow on Aristotelian phase space; interactions are as usual given by a social potential. This separation provides a principled way to tune inertial and collective effects independently, potentially allowing controlled acceleration or suppression of convergence by modifying the free Hamiltonian sector.

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