Matrix-product state skeletons in Onsager-integrable quantum chains

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Matrix-product state (MPS) skeletons are connected networks of local one-dimensional quantum lattice models with ground states that can be represented by an MPS with finite bond dimension. Such skeletons have previously been discovered in classes of free-fermion models. In this talk, I will discuss how equivalent skeletons underlie the phase diagrams of additional families of local models that are representations of the Onsager algebra: the N-state Onsager-integrable chiral clock models. Such models are interacting, and their analysis is therefore more complex than in their free-fermionic counterparts.

The Onsager algebra consists of generators $\{A_l, G_m | l, m \in \mathbb{Z}\}$, and the models we consider have Hamiltonians of the form $H = \sum_m t_m A_m$. From this, defining a Laurent polynomial $f(z) = \sum_m t_m z^m$, the skeleton can be shown to consist of models obeying the "square condition" $f(z) = z^p g(z)^2$, where p is an integer and $g(z) = \sum_{k=0}^{d} z^k$ is a finite polynomial. This condition is sufficient to guarantee the existence of an exact MPS ground state in the gapped regions of the phase diagram that smoothly connect to any fixed-point Hamiltonian A_m; however, it is not a necessary restriction. Outside of these gapped regions, this MPS remains an eigenstate, but is only the ground state within a particular symmetry sector. From our arguments, we can also construct certain excited states, going beyond the previous analysis of ground states along the $N = 2$ free-fermion MPS skeleton.

Moreover, I will demonstrate that the MPS skeleton is dense within the phase diagram. For any Hamiltonian H with Laurent polynomial $f(z)$, we can define $g(z) = \sqrt{z^{-p} f(z)}$. This series can be truncated at some finite power D. The Hamiltonian corresponding to the square of this polynomial has a ground state that approximates the ground state of H. Thus, we have an analytic method for approximating the ground state of any model in the family as an exact MPS.

Furthermore, as an application of this analysis, I will discuss how these results provide a pathway to calculating correlation functions such as the disorder parameter, which I will demonstrate for a simple example. Finally, I will discuss how this work can be utilised to understand the first-order phase transitions of the ground state into different symmetry sectors.