Energy Localization and Lyapunov Exponents in a Mean-Field Model without Linear-Dispersion

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In this talk, we focus on energy localisation and nonequilibrium phenomena in a mean-field model with purely non-linear dispersion. In the absence of linear dispersion, mean-field models exhibits strong localisation phenomena with cases where the initial excitation of a wave-packet will be almost preserved at all times [4]. For generic initial conditions, we find that the maximum Lyapunov exponent decays as a power-law in terms of the system size. For a fixed N size and varying the energy, we derive an analytic estimate on the Lyapunov exponent's upper-bound $\lambda(\varepsilon)$, where ε is the energy density. This law approximates the energy dependence in the strong nonlinearity regime.

In addition, we study the route to thermalization through the computation of the probability density distributions of the momenta of the system and their slow convergence into a Gaussian distribution in the context of non-extensive statistical mechanics and Tsallis entropy, a process that is further prolonged as the number of particles increases.

Finally, we observe that the maximum Lyapunov exponent decays as a power–law with respect to the system size, indicating "integrable-like" behaviour in the thermodynamic limit. Well-known systems displaying a similar behavior are the Mean - Field Hamiltonian (HMF) [1,2] and the Fermi–Pasta–Ulam–Tsingou model with long-range interactions (FPUT-LRI) [3].

[1] V. Latora, A. Rapisarda, and S. Ruffo, Lyapunov instability and finite size effects in a system with long-range forces, Phys. Rev. Lett., 80, 692–695 (1998). [2] C. Anteneodo and C. Tsallis, Breakdown of exponential sensitivity to initial conditions: Role of the range of interactions, Phys. Rev. Lett., vol. 80, pp. 5313–5316, (1998). [3] H. Christodoulidi, C. Tsallis and T. Bountis, Fermi-Pasta-Ulam model with long-range interactions: Dynamics and thermostatistics, Europhys. Lett., 108, 40006 (2014). [4] H. Christodoulidi, C.G. Antonopoulos, Energy localisation and dynamics of a mean-field model with non-linear dispersion, Physica D: Nonlinear Phenomena, Volume 471, 134432 (2025).