Fluctuation-response theory for nonequilibrium Langevin dynamics

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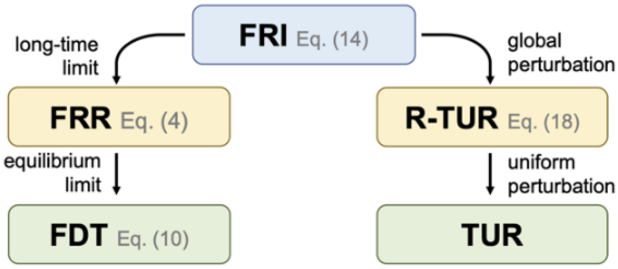

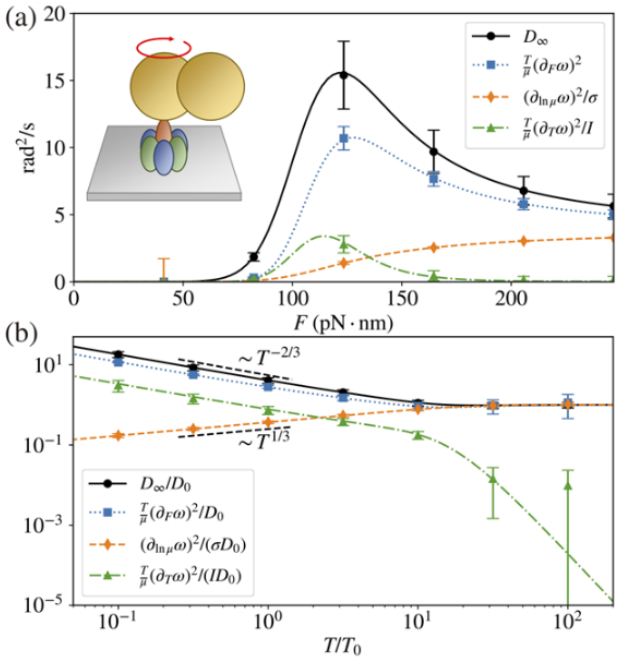

Understanding the relationship between fluctuations and response is a central problem in statistical physics. While the fluctuation–dissipation theorem (FDT) provides a complete description near equilibrium, its extension to nonequilibrium systems remains an open challenge. At the same time, thermodynamic and kinetic uncertainty relations (TUR and KUR) have revealed fundamental trade-offs between fluctuations, dissipation, and dynamical activity, but their connection to response theory has remained unclear. Clarifying this connection is essential for developing a unified description of nonequilibrium fluctuations and their physical constraints.

In this work, we establish a unified fluctuation–response relation (FRR) for nonequilibrium Langevin dynamics, thereby bridging these two major directions. By exploiting a common mathematical structure underlying fluctuations and responses of empirical density and current, we derive an exact identity that connects global fluctuations of trajectory-dependent observables to their local responses under perturbations in force, mobility, and temperature. This identity generalizes the equilibrium FDT to nonequilibrium steady states and recovers it in the appropriate limit. It applies broadly to a wide class of observables defined along stochastic trajectories.

Building on this result, we further derive finite-time fluctuation–response inequalities (FRIs) using a functional Cramér–Rao bound. These inequalities extend the fluctuation–response connection beyond the steady-state and long-time limit, making them applicable to arbitrary initial conditions and finite observation times. From the FRI, we obtain response uncertainty relations that provide experimentally accessible bounds by relating response amplitudes to fluctuations through global perturbations. In contrast to exact identities, these inequalities offer practical constraints that can be tested in realistic experimental and numerical settings. This framework reveals a hierarchical structure linking FRR, FRI, FDT, and TUR within a single theoretical scheme.

As an application, we analyze the F1-ATPase molecular motor model, where enhanced diffusion arises near critical driving. We demonstrate that the derived response-based bounds tightly constrain the long-time diffusion coefficient and capture its scaling behavior across different perturbation protocols. In particular, the bounds track the characteristic enhancement of diffusion near the critical regime. These results highlight the practical relevance of response-based bounds in nonequilibrium systems.

Our work provides a unified perspective on fluctuation–response relations in continuous-state systems and establishes a general framework connecting fluctuation theorems, response theory, and uncertainty relations in nonequilibrium statistical physics.

Reference: H. M. Chun, E. Kwon, H. Park, and J. S. Lee, arXiv:2601.16387