Firing rate estimates for a noisy FitzHugh-Nagumo neuron

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Neural excitability is commonly modeled through highly nonlinear differential equations or discrete nonlinear maps, simulating the complex dynamics of membrane potentials in neural cells. While such formulations are necessary to capture biologically realistic behavior, their strong nonlinearity poses significant analytical and interpretative challenges, particularly when extended to networked systems, and in the presence of a noisy environment. In the present work we focus on a FitzHugh-Nagumo (FHN) neuron subject to a noisy input in the variable that represents the membrane potential, and we derive an analytical expression for the Interspike Interval (ISI) probability distribution. The relevance of this distribution rests in the fact that, together with its average, i.e. the reciprocal of the firing rate, it represents one of the most easily measurable quantities in actual neuroscientific measurement campaigns.

Our approach is based on the usage of an ad-hoc adiabatic approximation, where the firing process is formalized as a Kramers escape problem with well depth self-consistently modulated by the time elapsed since the last neuron activation. The analytical expression is consistent with simulations, up to a correction term made necessary by effects which go beyond the additive noise regime assumed in the Kramers approximation, due to interactions between the fast and the slow time scale present in the FHN dynamics. Once these effects are taken into account, we use the explicit functional form to construct a simplified discrete dynamical system that reproduces the ISI statistics of a single nonlinear FHN neuron, showing good correspondence.

With the final goal of going towards studies related to the information processing capabilities of networks based on realistic neurons, we proceed to devise a minimal scheme for the construction of networks of coupled simplified neurons, by considering the alterations induced by an external input in the resting-state well depth considered in the Kramers approximation. We then compare the collective states that arise on directed networks of FHN neurons with those that are observed on networks of coupled simplified units on the same network structure. For aptly chosen parameter values, the emergent states appear to be qualitatively similar, but further work is needed to make the correspondence more solid for quantitative observables as well.