

Stochastic geometry of fiber intersections: effects of directional bias and dimensionality reduction

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In the study of soft matter and physical networks, ranging from biological tissues and biopolymer gels to non-woven synthetic fabrics, macroscopic bulk material properties are fundamentally dictated by their microscopic architecture [1]. Features such as mechanical elasticity, non-linear stress response, and fluid transport capabilities are heavily influenced by the spatial distribution, connectivity, and density of fiber intersections, which act as critical force-bearing nodes and topological cross-links [2]. To elucidate how statistical orientation rules govern these emergent intersection patterns, we investigate 3d fiber networks simulated as straight lines confined in a cubic field of view.

Specifically, we model the fiber networks using isotropic Poisson line processes as well as anisotropic von Mises-Fisher distributions confined to a bounded 3D cubic domain. After extracting the spatial point patterns of pairwise filament intersections, we characterize network non-uniformity using the Nearest Neighbor Index (NNI) [3,4], a classical measure of clustering relative to complete spatial randomness.

To bridge theoretical spatial statistics with experimental microscopy, we perform three complementary structural analyses: (i) global 3D point pattern statistics, (ii) depth-limited 2D cross-sections that emulate scanning electron microscopy (SEM) imaging, and (iii) full-depth 2D planar projections. Our global 3D analysis reveals that fiber intersections inherently cluster in space. Furthermore, we find that the von Mises-Fisher concentration parameter exerts control over this clustering behavior. Our projection analyses demonstrate that mapping 3D network structures to 2D planes introduces “pseudo-clustering” artifacts, highlighting the geometric consequences of finite focal depths in real images obtained from microscopy.

This study establishes a quantitative link between fiber alignment and intersection statistics, providing a physical modeling framework for interpreting the spatial architecture of complex fibrous materials.

References:

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