

Thermal Hofstadter Butterflies

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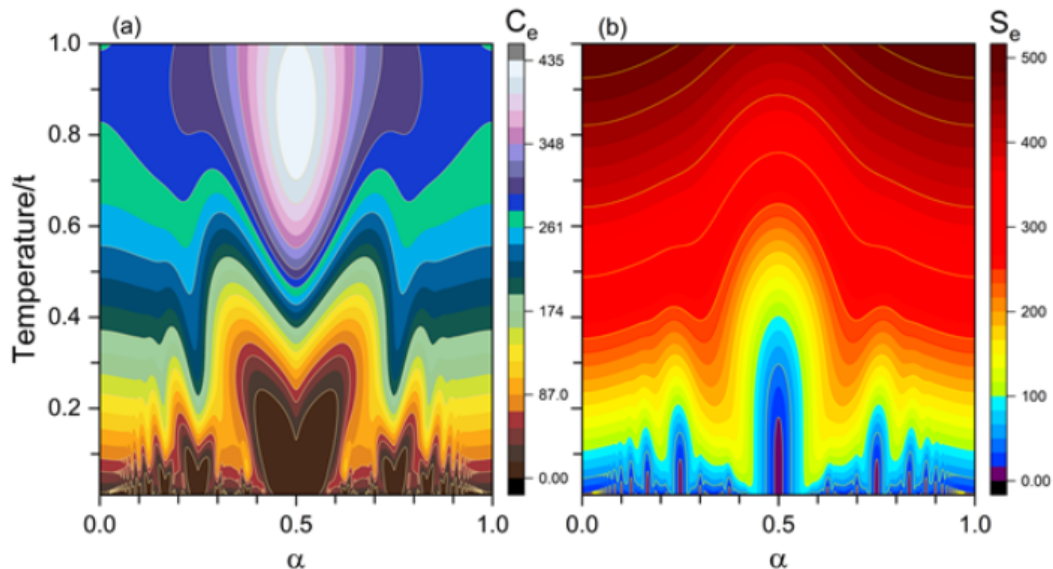
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Fractal electronic spectra arising from the competition between lattice periodicity and magnetic flux are a fundamental hallmark of two-dimensional (2D) quantum systems, famously known as the Hofstadter butterfly for electrons under a magnetic field in a 2D square lattice [1]. Despite extensive studies of its spectral properties and topological classification, the thermodynamic response of the Hofstadter butterfly's fractal energy landscape has remained largely unexplored. Through the use of single-orbital tight-binding equations in connection to Fermi-Dirac statistics, we perform an original characterization of the thermal behavior for electrons as a function of magnetic flux α and temperature T by calculating the electronic entropy S_e , and specific heat C_e , at half-filling, for square, honeycomb, and triangular lattice geometries.

We find that the thermodynamic response reflects universal thermal effects for the three lattices around magnetic flux $\alpha = 1/2$, as well as unique features at particular α numbers for each lattice geometry. Both thermal quantities reveal fractal signatures in each spectrum through intricate patterns arising from the coupling between α and T . We identify striking self-similarity in S_e and C_e , tracing heart-shaped specific heat and tunnel-like entropy contours that repeat at specific lattice-dependent magnetic fluxes. The figure shows these patterns for the specific heat in panel (a), and entropy in panel (b) in the square lattice.

Entropy minima at low temperatures play a remarkable role, acting as fingerprints for the butterfly spines, resolving the underlying fractal spectra. This specific low- T limit as α is varied can serve as an alternative window into the fractal nature of the spectra. Specifically, entropy minima occur at characteristic flux $\alpha = 1/2, 1/4, 1/6, \dots$, for the square lattice, and $\alpha = 1/2, 1/3, 1/4, \dots$, for the honeycomb lattice. Our calculations would help identify these repetition patterns where thermal fractal signatures emerge, allowing one to indirectly 'observe' the underlying butterfly spectrum by looking for entropy minima, and assist in comparison with experiments.

These findings may establish thermal measurements as high-resolution spectroscopic probes, providing a robust framework for recognizing fractal signatures through thermodynamic analysis across diverse nanostructures [2].



References:

- [1] Hofstadter, D. R. Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields. *Phys. Rev. B* 1976, 14, 2239–2249.
- [2] Cortés, N., Castorene, B., Peña, F. J., Melo, D., Ulloa, S. E., and Vargas, P. arXiv:2603.07424.