From data to stochastic dynamics: nonlinear Langevin modeling of asymmetric persistence in meteorology

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A number of econometric studies have demonstrated asymmetric cross-correlations between different markets and financial assets, indicating nontrivial directional dependencies [1]. The existence of asymmetric autocorrelations has been shown in two non-stationary natural time series—the sunspot and temperature sequences — and a modified DFA method, separating positive and negative trends, has been proposed [2]. However, some hidden memory mechanisms generating asymmetric autocorrelations may also be present in stationary processes, where trend episodes are not directly observable or easily identifiable.

Here we introduce a nonlinear stochastic model of the Langevin type that incorporates finite-range asymmetric persistence. In this framework, persistence represents a simplified form of memory: instead of considering the last m states, only the signs of the previous m increments are taken into account. Our model differs from well-known fractional models in one more fundamental aspect—persistence in our case can be asymmetric.

For example, to incorporate asymmetric persistence of length $m = 1$ into the standard Langevin framework, we introduce an additional factor c, defined as a function of the last increment, which controls the sign of the diffusion term. If the last increment is positive, then $c = 1$ with probability p (and $c = -1$ with probability $1 - p$). Conversely, if the last increment is negative, then $c = -1$ with probability q (and $c = 1$ with probability $1 - q$). The parameters p and q characterize the asymmetric persistence associated with upward and downward movements, respectively. Depending on the choice of the drift and diffusion functions, as well as the values of the persistence parameters, the model can generate either stationary or nonstationary time series.

For the special case of $m = 1$, the model is complemented by a procedure for reconstructing the drift and diffusion functions directly from stationary time series data. This procedure has a hybrid character: it employs a standard numerical method for reconstructing the classical Markovian Langevin equation, while introducing appropriate analytical corrections is related to the effect of asymmetric persistence. An important part of the procedure is the proposed method for estimating the persistence parameters.

We evaluated the procedure using stationary time series generated by our model, considering multiple linear and nonlinear configurations of the drift and diffusion functions, as well as different values for the persistence parameters p and q. In every scenario, the method successfully recovered the original forms of the drift and diffusion functions.

The methodology was implemented on a meteorological time series, namely the daily air pressure record from Maastricht spanning 1906–2025. The inferred persistence parameters are markedly asymmetric and exhibit antipersistent values ($p = 0.42$, $q = 0.25$). Nonlinear drift and diffusion functions were reconstructed, delineating the stochastic dynamics underlying the examined meteorological process. Time series synthesized using the derived Langevin-type framework reproduce stochastic signatures fully consistent with those observed in the original empirical dataset.

References:

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[2] J. Alvarez-Ramirez, E. Rodriguez, J. C. Echeverria, Physica A, 388, 2009.