Fundamental temperature in the superstatistical description of multi-particle kappa distributions

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Ions and electrons in collisionless plasmas out of equilibrium commonly exhibit velocity distributions that are not Maxwellian. Among them, probably the most successful model is the kappa distribution, where the Boltzmann exponential factor is replaced by a power law with exponent κ, sometimes called the spectral index. In the limit $\kappa \to \infty$ the Maxwellian distribution of equilibrium thermodynamics is recovered, while for small values of the spectral index the systems have high degrees of interparticle correlations and larger distance from thermal equilibrium. The origin of the kappa distribution in these systems out of equilibrium is usually explained by two generalizations of statistical mechanics, namely Tsallis' non-extensive statistical mechanics and, more recently, Beck and Cohen's superstatistics.

In the theory of superstatistics, the inverse temperature $\beta = 1/(k_B T)$ of the canonical ensemble is promoted to a random variable, having its own probability density, and in the superstatistical reconstruction of the kappa distribution, the uncertainty about temperature is reflected in the value of the spectral index. However, the observability of temperature in superstatistics is not, in general, guaranteed: temperature is not given by the value of an observable function of the degrees of freedom of the system (unlike the energy, which is always the value of the Hamiltonian), and thus has to be statistically inferred. Furthermore, in Bayesian interpretations of superstatistics, the inverse temperature does not actually fluctuate, but is an unknown or uncertain parameter. In this work, these issues are illustrated by exploring the superstatistical formulation of the N-particle kappa distribution and connecting the distribution of the superstatistical inverse temperature with that of the recently proposed fundamental inverse temperature, a model-dependent function of the energy that seems to play an important role in the properties of superstatistical systems. Regarding this function, a new important theorem is presented that connects expectation values of functions of the superstatistical inverse temperature with those of transformed functions of the fundamental inverse temperature, providing new insights into the nature of temperature in the superstatistical framework, as well as practical tools for the computation of statistical properties of systems described by N-particle kappa distributions.