Learning dynamics and cascade of phase transitions in the Restricted Boltzmann Machine

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Generative models are classic Machine Learning frameworks and, as highlighted by the last Nobel Prize in Physics, a fertile area for physicists. Following this path, we study Energy-Based Models, a central concept in generative modeling, and demonstrate how critical phase transitions naturally emerge during training. Specifically, we examine feature encoding in a prototypical energy-based generative model, the Restricted Boltzmann Machine (RBM). Starting with an analytical investigation using simplified architectures and data, we conclude with numerical analysis of real training on real datasets. Our study tracks the evolution of the model's weight matrix via singular value decomposition, revealing a series of phase transitions linked to the progressive learning of the principal modes of the empirical probability distribution. Initially, the model learns the center of mass of the modes and gradually resolves all modes through a cascade of phase transitions. We first describe this analytically in a controlled setup that allows precise study of the training dynamics. We then validate these findings by training Binary-Binary RBMs on real datasets. Using datasets of increasing dimensionality, we confirm that learning indeed induces sharp phase transitions in the high-dimensional limit. Additionally, we propose and test a mean-field finite-size scaling hypothesis, demonstrating that the first transition belongs to the same universality class as our analytical results, reminiscent of the mean-field paramagnetic-to-ferromagnetic phase transition.