Number of local minima in discrete-time fractional Brownian motion: non-Gaussian fluctuations driven by long-range memory

Maxim Dolgushev[1]

[1]Sorbonne University, Paris, France

The number of local minima is among the simplest geometric observables that can be extracted from a fluctuating signal, yet it already encodes detailed information on the underlying dynamics. This contribution addresses the statistics of the number of local minima in discrete-time fractional Brownian motion (fBm), a non-Markovian Gaussian process with stationary increments and long-range temporal correlations.

Whereas the average number of local minima grows linearly with the signal length and remains largely blind to long-range memory, the fluctuations display a sharp transition at the Hurst exponent H=3/4. Below this threshold, a Gaussian central-limit regime persists after suitable rescaling. Above it, temporal correlations qualitatively reshape the fluctuations: they are no longer Gaussian and instead fall into the Rosenblatt universality class.

This transition is shown through a Hermite-Wick decomposition of the local-minimum indicator, which isolates the term responsible for the anomalous large-scale behavior induced by the long-range memory of fBm. The analysis yields the full minima-counting process, thereby providing its complete multi-time statistics. Below H=3/4, the process counting the number of local minima converges to Brownian motion, whereas above H=3/4, it converges to the Rosenblatt process. This is illustrated by the two-time covariance of the minima-counting process, which exhibits two qualitatively distinct regimes, matching Brownian and Rosenblatt covariances on either side of the threshold.

Reference:

M. Dolgushev and O. Bénichou, Phys. Rev. Lett. 136, 117101 (2026)