

Quantitative Interaction Laws for Discrete Breathers

Chengguan Fang¹, Ling Deng¹, Weicheng Fu^{2,1}, Yisen Wang¹, Liang Huang¹, Sergej Flach^{3,4,5}

¹Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, and Key Laboratory for Magnetism and Magnetic Materials of MOE, Lanzhou University, Lanzhou, China, ²Department of Physics, Tianshui Normal University, Tianshui, China, ³Center for Theoretical Physics of Complex Systems, Institute for Basic Science, Daejeon, Republic of Korea, ⁴Basic Science Program, Korea University of Science and Technology, Daejeon, Republic of Korea, ⁵Centre for Theoretical Chemistry and Physics, The New Zealand Institute for Advanced Study, Massey University Albany, Auckland, New Zealand

Discrete breathers (DBs) are spatially localized vibrational modes ubiquitous in physical systems. Despite decades of research, their mutual interactions represent a fundamental yet unexplored facet of nonlinear dynamics. Here we uncover quantitative interaction laws for DB pairs, encoded in tunable energy beating whose amplitude provides a direct coupling metric shaped by energy mismatch, phase configuration, and a separation yielding exponentially decaying, parity-sensitive coupling. The coupling can be strongly modulated by frequency locking, which emerges only under restricted excitation conditions. A minimal nonlinear dimer encapsulates all salient behaviors across diverse lattice models. These results promote DBs from static energy traps to versatile elements for engineering energy transport, tailoring vibrational spectra, and generating acoustic frequency combs.