

On the universality classes of tetracritical systems with nonlinearly realized $SO(N)$ symmetries

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There exists a plenty of real physical systems exhibiting tetracritical behavior. However, complete systematization of their universality classes, unlike bi-critical systems, is still lacking. A possible step to fill this gap is suggested basing on an unconventional nonlinear sigma-models (NLSMs) – namely Stiefel sigma-models. A simpler examples of NLSMs are well-known (see e.g. [1]), these involve single coupling constant, and describe bi-critical systems only. Situation with Stiefel sigma-models is principally different, as it was shown in [2]. Within the NLSMs on real Stiefel manifolds $SO(N)/SO(N - k)$, using one-loop renormalization and the analysis of scaling behavior, a new infinite series of universality classes of tetra-critical systems with orthogonal symmetry have been derived [3]. The Stiefel NLSMs incorporate orthogonal and Grassmann subsystems of Goldstonions, what implies two coupling constants (or competing order parameters). The universality classes in that series are identified by the respective (tetra)critical exponents and depend on N , k and $\epsilon = d - 2$. These findings are of basic novelty and suggest theoretical groundwork that could be very helpful for better understanding and systematizing diverse complex phenomena, including possibly the realm of high-temperature superconductivity.

References:

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