A Long Prism Geometry for Spin Glasses: Intensive simulations and the lower critical dimension

Massimo Bernaschi[1], Luis Antonio Fernández[2], **Isidoro González-Adalid Pemartín**[1], Víctor Martín-Mayor[2], Giorgio Parisi[3,4,5,6], Federico Ricci-Tersenghi[4,5,6]

[1]CNR - Istituto Per Le Applicazioni Del Calcolo, Rome, Italy, [2]Departamento de Física Teórica - Universidad Complutense de Madrid, Madrid, Spain, [3]International Research Center of Complexity Sciences, Hangzhou International Innovation Institute, Beihang University, Hangzhou, China, [4]Dipartimento di Fisica - Sapienza Università di Roma, Roma, Italy, [5]CNR-Nanotec, Rome Unit, Roma, Italy, [6] INFN, Sezione di Roma, Roma, Italy

Dimensionality in spin glasses (and more generally in complex systems) plays a fundamental role in determining critical phenomena. In particular, the lower critical dimension, D_{lc}, marks the lower limit at which an ordered phase can exist at finite temperature [1]. In the Ising Edwards–Anderson model in the absence of a field, several theoretical frameworks exist, such as the Replica Symmetry Breaking (RSB) and Droplets models, which, despite different physical pictures, predict a lower critical dimension close to 2.5 [1–4]. Determining D_{lc} requires analytical and numerical effort and is a central problem in spin-glass physics.

In this work, building on the analytical arguments presented by Franz, Parisi, and Virasoro [2], I present a general computational strategy that uses an elongated rectangular prism in three dimensions to determine D_{lc}. In this quasi-one-dimensional system, correlations along the prism axis, $\xi_{\|}$, decay exponentially with the system's transverse size, L. In particular, RSB predicts a decay that scales as $L^{4/3}$. This prediction, obtained from the study of low-energy excitations at low temperature, directly reflects $D_{lc} \simeq 5/2$.

In this framework, determining D_{lc} becomes a finite-size scaling problem, which we verify using large-scale GPU-based Monte Carlo simulations on systems with up to $24^2 \times 88$ lattice sites and more than 400,000 hours of computation. To obtain these results, it was necessary to consider several relevant technical aspects: first, the unexpected efficiency of Houdayer's clustering algorithm [5] in this geometry, which drastically accelerates thermalization, and the use of open boundary conditions along the prism's longitudinal axis, which reduces finite-volume effects and allows extraction of $\xi_{\|}$ from significantly shorter systems.

Our numerical results, obtained by studying the correlations of odd observables such as the plane overlap, $Q(z)$, are consistent with $D_{lc} = 5/2$, providing direct numerical evidence that both the RSB and Droplets scenarios describe this quasi-one-dimensional regime [6]. However, this work also opens the door to a possible comparison of the two theories, since when considering correlations of even observables, such as $Q(z)^2$, the two theories differ in their predictions, which could allow us to determine which better captures the nature of correlations in spin glasses.

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