Transport properties in a model of confined granular mixtures at moderate densities

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Confined granular mixtures—such as grains of different sizes or masses fluidized in vertically vibrated, quasi-two-dimensional geometries—are a paradigmatic class of driven nonequilibrium systems. Despite their relevance for both fundamental statistical physics and segregation phenomena, a predictive kinetic theory at finite density is still incomplete: previous results for confined mixtures were mostly restricted either to the dilute regime or to tracer limits. In this contribution, I present a theoretical study that closes part of this gap by developing the Navier–Stokes description of a moderately dense, confined granular mixture with arbitrary composition.

The analysis is based on the collisional -model, a coarse-grained description that captures the transfer of energy from vertical vibration to the horizontal motion of the grains, together with the inelastic revised Enskog kinetic equation. Starting from this framework, I derive the hydrodynamic balance equations for mass, momentum, and energy, and obtain the corresponding constitutive relations by means of a Chapman–Enskog expansion to first order in spatial gradients. As in ordinary dense gases, the transport coefficients are formally determined by a coupled set of linear integral equations. Their explicit solution is highly nontrivial for mixtures, since the number of unknown transport coefficients increases rapidly with the number of species and depends on concentration, restitution coefficients, mass ratio, size ratio, and density.

To obtain workable expressions, I focus on the physically relevant steady state and solve the problem within a leading Sonine-polynomial approximation. This yields explicit analytical forms for the diffusion transport coefficients associated with the mass flux, as well as for the shear and bulk viscosities, in particular for binary mixtures in quasi-two-dimensional conditions. The theory extends previous dilute and tracer-limit results and shows how collisional dissipation, density, and compositional asymmetry jointly control momentum and mass transport. An important outcome is that the effect of inelasticity and density on transport is generally weaker than in the conventional inelastic hard-sphere model, highlighting the distinctive character of confined, vibrated systems.

As a direct application, I use the derived diffusion coefficients to compute the thermal diffusion factor and establish criteria for segregation under the combined action of gravity and temperature gradients. The resulting phase diagrams identify the transition between Brazil-nut and reverse Brazil-nut behavior and reveal that moderate-density effects can substantially shift the segregation boundaries depending on the driving conditions. Overall, this work provides a unified theoretical framework for transport and segregation in confined granular mixtures at moderate densities, and offers a solid basis for future comparison with simulations and experiments, as well as for stability analyses of the homogeneous steady state.