# Frequency-Resolved Criticality Metrics from EEG Data: Effective Coupling and Effective Dimensionality in Healthy versus Concussed Adolescents

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In modern neuroscience, the critical brain hypothesis states that neural networks operate near a phase transition, where functional capabilities such as optimal information processing and flexible neural coordination naturally emerge. Neural dynamics can be modeled as a multivariate Ornstein–Uhlenbeck process driven by Gaussian white noise, where the drift matrix encodes synaptic interactions between neurons.

In this framework, an independent and identically distributed (i.i.d.) Gaussian connectivity structure in the large-$N$ limit serves as a generic and asymmetric theoretical baseline [1, 3].

Since brain activity is characterized by rich spatiotemporal structure, including diverse rhythmic oscillations that coexist with broadband $1/f$ noise, time-integrated analyses often fail to capture the brain's intricate temporal organization [4]. We therefore focus on the frequency domain and analyze the spectral density matrix (SDM), that is, the frequency-resolved covariance matrix of the recorded brain signals.

We study EEG signals from 32 healthy and 15 concussed male adolescents [2]; the latter have been diagnosed with mild traumatic brain injury (mTBI). The data for each individual represent multivariate time series of dimension $N \times 27$, where $N \approx 80{,}000$ is the length of the recording, and 27 the number of sensor locations. We extract two real-valued, frequency-dependent metrics from the spectral density matrix. The first, known as effective dimensionality (ED), is a model-free measure of network coordination based on the participation ratio of the SDM eigenvalues. The second, known as effective coupling (EC), quantifies the proximity to criticality by fitting the SDM eigenvalue spectrum to the theoretical distribution of the large-$N$ i.i.d. Gaussian model [1]. We compute both metrics as functions of frequency using resting-state sensor-space electroencephalography (EEG) data (sampled at 250 Hz). We discuss the implications of these frequency-specific signatures for the critical brain hypothesis and their potential as dynamical markers in clinical concussion assessment.

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