

# Long-Range Hopping in Quantum Bosons: From Non-BKT Criticality to Excitonic Supersolids

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We present exact large-scale Quantum Monte Carlo results for the zero-temperature phase diagram of the one-dimensional Bose–Hubbard model with power-law hopping decaying with distance as  $1/r^\alpha$ . This model provides a minimal framework for investigating how long-range kinetic processes qualitatively modify the physics of interacting bosons in low dimensions, beyond the conventional short-range paradigm. Our focus is on the superfluid–Mott-insulator transition at unit filling and on the nature of the superfluid phase itself.

We find that, for all  $1 < \alpha \leq 3$ , the transition is continuous and scale invariant, in a manner incompatible with the standard Berezinskii–Kosterlitz–Thouless (BKT) scenario of the short-range one-dimensional Bose–Hubbard model. The BKT behavior is recovered only for  $\alpha > 3$ , thereby identifying  $\alpha = 3$  as the threshold beyond which the system crosses over to short-range critical behavior. We characterize the resulting universality class through finite-size scaling of winding-number fluctuations, data-collapse analysis, and a careful examination of the low-energy excitation spectrum. Taken together, these results demonstrate that long-range hopping alone can induce unconventional criticality in one-dimensional bosonic systems.

In addition to altering the critical point, long-range hopping also qualitatively reshapes the superfluid phase. For  $\alpha \leq 2$ , the ground state exhibits true long-range order, indicating behavior more closely analogous to that of higher-dimensional bosonic systems. By contrast, for  $2 < \alpha \leq 3$ , we identify an intermediate regime characterized by anomalous quasi-long-range order, distinct from both conventional one-dimensional Luttinger-liquid behavior and higher-dimensional condensation. These findings reveal a rich internal structure within the weak long-range regime and provide exact numerical benchmarks for ongoing theoretical efforts aimed at understanding long-range quantum matter.

More broadly, this work contributes to growing evidence that one-dimensional bosonic systems can exhibit scale-invariant phase transitions beyond the standard BKT paradigm when suitably extended couplings are present. The results are directly relevant to current experimental platforms in which long-range hopping or effective long-range couplings may be engineered, including trapped ions, dipolar quantum gases, and excitonic lattice systems.

We also briefly connect these ideas to collaborative numerical work on dipolar excitons in a lattice, where long-range hopping, combined with dipolar interactions, supports excitonic supersolid behavior at fractional fillings. In that setting, extended kinetic processes and interactions cooperate to stabilize phases exhibiting both density order and phase coherence, thereby providing an experimentally relevant realization of the broader many-body phenomena discussed in the first part of the talk.