Nonequilibrium criticality in the dynamics of synchronizing oscillators

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The study of synchronous dynamics has mostly focused on the transition to synchronization from a static (steady-state) perspective, while the dynamical process whereby systems of oscillators synchronize at long times has received much less attention. Most likely, this is due to the natural expectation that this process should be strongly system-dependent. Surprisingly, that is not at all the case. As we have recently shown in a series of publications [1-5], the synchronization process displays robust universal features previously observed in the context of nonequilibrium critical dynamics. This was suggested by a mathematical connection between spatially discrete oscillator models and the continuum equations of surface kinetic roughening, and has been thoroughly confirmed by means of detailed numerical studies of 1D systems of phase [1,3,4,5] and some limit-cycle oscillators [2,3] in the presence of quenched disorder (a random assignment of intrinsic frequencies) [1,2,5], time-dependent noise [3,5], or a combination of them [4]. Taken together, these explorations provide strong evidence confirming that the synchronization process in these systems is characterized by forms of generic scale invariance associated with kinetically rough interfaces, such as that of the Kardar-Parisi-Zhang (KPZ) universality class under time-dependent or columnar disorder. In fact, the precise form of the coarse-grained dynamical equations and the role of symmetries and randomness can be analytically understood by a combination of continuum approximations and phase-reduction methods [2,4], and the existence of well-known crossover and finite-size effects [5]. We moreover find that fluctuations around the average growth appear to generically follow a Tracy-Widom distribution, frequently associated with the KPZ nonlinearity. Synchronization and surface growth, two important branches of contemporary statistical and nonlinear physics, are thus more closely related than previously anticipated due to such robust universal features, which make the experimental observation of the nonequilibrium criticality of synchronization an alluring possibility [5]. Some preliminary results on phase oscillators in lattices of higher dimensions and chains of chaotic oscillators, will also be discussed.

References:
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