A Statistical Mechanics Approach to Immiscible Two-Phase Flow in Porous Media

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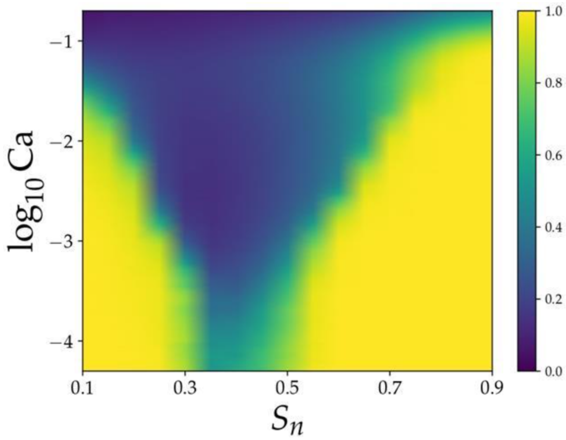

The simultaneous flow of immiscible fluids in a porous medium is a complex problem at the pore scale where viscous and capillary forces complete while being constrained by the pore structure of the material in which the fluids move. On the scale at which the porous medium appears, the Darcy scale, the same flow has the character of being a single fluid which obeys a complex rheology.

The Darcy scale constitutive equation for the flow relates flow rate to pressure gradient. It is possible to identify four flow phases depending on the pressure gradient [1]. At very low pressure gradients, the capillary forces hold the fluid-fluid interfaces in place so that the flow appears through open channels spanning the porous medium and surrounded by stuck fluid clusters (phase 1a). As the pressure gradient is increased, the fluid-fluid interfaces start moving, leading to changes in the shape of the stuck clusters. However, the flow is still dominated by the open channels (phase 1b). In both cases, we find that the flow rate is proportional to the pressure gradient. Further increasing the pressure gradient leads to previously stuck fluid clusters increasingly being mobilized and hence participate in the transport of the fluids. The flow rate is in this phase proportional to the pressure gradient raised to a power somewhat below two (phase 2). As the pressure gradient is raised even further, we reach at some point that all clusters that can move will do so. The constitutive equation then reverts to a linear relation between flow rate and pressure gradient (phase 3).

Recently, using Boltzmann machine learning, the flow problem has been mapped onto an Ising spin model where all spins are interacting directly with each other [2]. We show in the figure the Edwards-Anderson order parameter for the spin model as a function of the flow parameters, Ca which is the capillary number, a dimensionless ratio between the viscous and the capillary forces, and the Sn, the saturation which is the fraction of pore space occupied by one of the fluids. We see in the figure that there is a spin glass phase and a paramagnetic phase separated by a critical line. It turns out that this critical line coincides with the transition between phases 1b and 2 described above. The other transitions, 1a to 1b and 2 to 3 are crossovers.

Critical lines indicate and underlying statistical mechanics and therefore also a thermodynamics on large scales. We present such a thermodynamics for immiscible two-phase flow in porous media [3].

References:

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[3] A. Hansen and S. Sinha, Immiscible two-phase flow in porous media: a statistical mechanics approach, arXiv:2603.09658; DOI:10.48550/arXiv.2603.09658.