# Thermodynamic Uncertainty Relation for Non-Gaussian Lévy-Type Active Matters with Time-Dependent Driving

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We investigate the extent to which thermodynamic uncertainty relations (TURs) retain their universal structure in overdamped stochastic systems subject to non-Gaussian, Lévy-type active fluctuations and driven by fully time-dependent protocols. TURs provide fundamental bounds that connect the precision of fluctuating currents to entropy production in nonequilibrium systems. While these relations are well understood for systems driven by Gaussian thermal noise under steady or weakly time-dependent conditions, their validity and structure in more general active and strongly driven settings remain less explored.

In this work, we demonstrate that a generalized TUR can be formulated for overdamped dynamics with non-Gaussian active noise by carefully accounting for the combined effects of temporal driving and stochastic fluctuations. A key step in our approach is the identification and incorporation of a dynamical response term, which captures variations arising both from changes in the observation time and from the speed of the external driving protocol. This term plays a central role in restoring a TUR-like bound that remains applicable even in the presence of heavy-tailed noise statistics and explicitly time-dependent control parameters.

The resulting generalized TUR applies broadly to arbitrary state-dependent observables and integrated current-like quantities, extending beyond conventional formulations restricted to stationary processes. Furthermore, we derive an explicit saturation criterion that clarifies when the bound becomes tight. In particular, we show that observables that more accurately resolve entropy-producing fluctuations—by effectively tracking the underlying irreversible processes—approach the bound more closely. This provides a concrete link between the informational content of an observable and its thermodynamic efficiency.

To support our theoretical findings, we perform numerical simulations of overdamped particles driven by compound-Poisson noise in both linear and harmonic potentials under time-dependent protocols. These simulations confirm the validity of the generalized TUR and illustrate how deviations from Gaussian statistics and the presence of active fluctuations influence the precision–dissipation trade-off.

Overall, our results extend the applicability of thermodynamic uncertainty relations to a broad class of active-matter systems characterized by non-Gaussian noise and temporal driving. They also provide some new insight into how activity modifies nonequilibrium fluctuation–dissipation constraints, offering a more general framework for understanding precision limits in driven stochastic systems.