Two-loop calculation of the incompressible active matter model with white noise correlation

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Active matter denotes a class of non-equilibrium many-body systems whose microscopic constituents consume energy to generate persistent motion. This continual energy input breaks detailed balance and leads to emergent collective phenomena such as long-range correlations, pattern formation, and large-scale coherent motion. We present a perturbative renormalization group study of the incompressible Toner-Tu model of active matter with emphasis on two-loop corrections. The model is formulated as a stochastic hydrodynamic theory subject to an incompressibility constraint and mapped onto a field-theoretic representation using Martin-Siggia-Rose-Janssen-de Dominicis response functional. This formulation provides a systematic framework for analyzing fluctuation effects and scale dependence in non-equilibrium systems. The previous studies established the one-loop renormalization structure of the incompressible Toner-Tu model with white noise [1], while subsequent work generalized the noise correlator at leading order [2]. In contrast, the present analysis addresses the two-loop problem for the incompressible model with white noise, which to our knowledge has not been carried out before. At this order, new classes of diagrams appear whose evaluation requires careful treatment of arising tensor structures. Building on known one-loop renormalization results, we discuss the structure of the relevant two-loop Feynman diagrams and analyze their contributions to the renormalization factors entering the theory. Ultraviolet divergences are treated within a dimensional regularization combined with minimal subtraction scheme, allowing a consistent identification of renormalization constants and anomalous dimensions. These results provide higher-order corrections to beta functions which modify the renormalization group flow, provide corrections to scaling behavior, shift fixed points in parameter space, and refine the phase diagram describing collective motion in incompressible active matter. The calculation also provides a systematic test of the robustness of leading-order predictions by identifying which features of the large-scale behavior persist after inclusion of higher-order fluctuation effects. The obtained results represent a step toward higher-order field-theoretic descriptions of active matter and contribute to understanding universal behavior in non-equilibrium many-body systems as well as provide a technical basis for future multi-loop investigations of incompressible active matter within the perturbative renormalization framework.

References:

[1] L. Chen and J. Toner, New J. Phys. 17, 042002 (2015).

[2] D. Husz, Štúdium hydrodynamickej teórie aktívnej hmoty, Master's thesis, Pavol Jozef Šafárik University in Košice. Central Register of Theses and Dissertations (2024), https://opac.crzp.sk/?fn=detailBiblioForm&sid=0CD22148AAB768F4955F7D857C89