Generalized Euler and Gibbs-Duhem relationship for generalized homogenous systems

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The Gibbs-Duhem relationship represents a fundamental dependence between the thermodynamic state parameters and their conjugate variables and can be derived from the homogeneity of the entropy function via the Euler relationship. Although homogeneity is normally assumed in classical thermodynamics, it does not always hold. A prominent example can be found in black hole thermodynamics which is based on the Bekenstein-Hawking entropy that is proportional to the area of the black hole. For example, it is well known that for the black holes such as the Schwarzschild, Kerr-Newman, or Reissner-Nordstrom, the Bekenstein-Hawking entropy is not homogeneous but quasi-homogeneous function of state variables, and the Euler and Gibbs-Duhem relationships have a different form from the standard one, which has already been discussed in the past by several authors.

On the other hand, further developments in quantum field theory, and entropic and quantum gravity suggest that the Bekenstein-Hawking entropy should be corrected with an additional term, or to be transformed by an application of a non-linear mapping, to take into account thermal fluctuations and quantum effects. However, the resulting black hole entropies are neither homogenous nor quasi-homogenous, and the appropriate property for their characterization is not available, as well as the corresponding Euler and Gibbs-Duhem relationships.

In this work, we advance the previous studies, and we generalize the notions of homogeneity and quasi-homogeneity, and derive the corresponding generalized Euler and Gibbs-Duhem relationships. We thus provide a general framework that can characterize thermodynamic formalisms that are built on entropies that are neither homogenous nor quasi-homogenous. We applied the results to different types of black hole entropy corrections, such as logarithmic or exponential ones, and to alternative entropy forms such as Renyi, Sharma-Mittal, or Kaniadakis black hole entropies. As a result, we provided new Gibbs-Duhem and Euler relationships for these entropies.

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