From Group Entropies to Black Hole Radiation: A Thermodynamic Perspective

Petr Jizba[1], Henrik Jensen[2], Piergiulio Tempesta[3]

[1]FNSPE, Czech Technical University In Prague, Prague, Czech Republic, [2]Centre for Complexity Science and Department of Mathematics, Imperial College London, London, UK, [3]Facultad de Físicas, Universidad Complutense de Madrid, Madrid, Spain

I discuss thermodynamic features of group entropies, focusing on the stretched-exponential entropy $S_{\alpha,\gamma}$. This entropy is well suited to systems with sub- or super-extensive state-space scaling, such as black holes, as it provides an extensive and composable entropy framework. Analogous to how the Gibbs–Shannon entropy yields Clausius entropy, $S_{\alpha,\gamma}$ can be expressed in terms of state variables via the MaxEnt prescription.

I examine the zeroth and second laws, identifying the empirical temperature and, via Carathéodory's formulation, the corresponding absolute temperature. I show that the heat capacity of the black hole associated with $S_{\alpha,\gamma}$ is negative (in the canonical ensemble), and that the resulting Stefan–Boltzmann law for blackbody radiation forms a two-parameter family of solutions. I will also discuss further implications for black hole thermodynamics.