Structural Phase Transitions in Evolving Networks Under Different Node-Deletion Mechanisms

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Many real-world networks evolve under concurrent processes of growth and contraction. Preferential attachment generates scale-free degree distributions with fat tails and hubs [1], yet empirical studies of declining networks show that contraction does not necessarily destroy structural organization: some shrinking networks maintain stable truncated degree distributions over long periods [2]. This raises a fundamental question: how does node deletion interact with preferential attachment to determine whether hubs persist, are suppressed, or disappear?

We study models in which, at each time step, a node is added with probability P_{add} together with m edges, or deleted with probability $P_{\text{del}} = 1 - P_{\text{add}}$. The balance between growth and contraction is measured by $\eta = P_{\text{add}} - P_{\text{del}}$, where $\eta > 0$ corresponds to net growth and $\eta < 0$ to net contraction. We analyze three growth-contraction mechanisms, each leading to a distinct stationary degree distribution $P(k)$.

1. Random Attachment + Random Deletion (RARD): In this case, deletion does not target hubs. For $-1 < \eta < 1$, the stationary degree distribution is Poisson-like [3]. At $\eta = 1$, it becomes exponential, indicating a structural transition in the pure-growth limit.

2. Preferential Attachment + Random Deletion (PARD): The stationary degree distribution exhibits a power-law tail for $\eta > 0$, while for $\eta < 0$ it becomes exponential. Thus a structural phase transition occurs at $\eta = 0$ [4].

3. Preferential Attachment + Preferential Deletion (PAPD): Here deletion is degree-biased and directly counteracts hub formation. The stationary degree distribution is power-law only in the pure-growth limit $\eta = 1$ and becomes exponential for any $\eta < 1$. Moreover, defining $\eta_c(m) = -\frac{m-2}{m+2}$, one finds that for $\eta < \eta_c(m)$ no stationary state exists and the network collapses to isolated nodes [5].

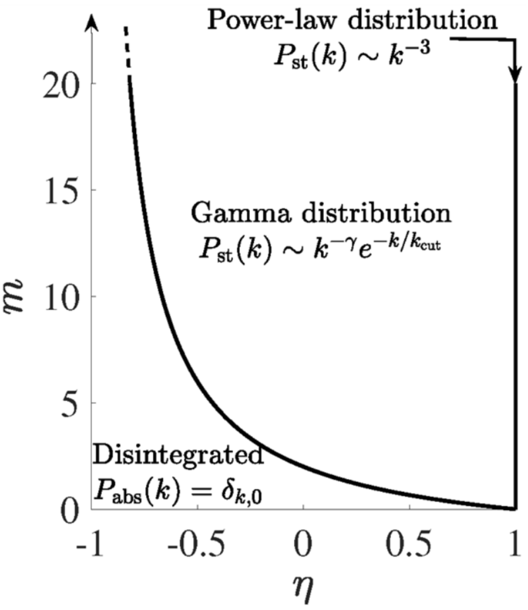

Figure 1. Phase diagram of the PAPD model showing the transition line $\eta_c(m) = -\frac{m-2}{m+2}$, separating the stationary and collapsing regimes.

These results show that the deletion mechanism is structurally decisive. Random deletion preserves the preferential amplification of high-degree nodes, allowing scale-free structure to persist when growth dominates. Preferential deletion, by contrast, acts as an anti-preferential-attachment force that suppresses hubs even under net growth. This provides a theoretical explanation for truncated or exponential degree distributions observed in transient networks such as dating platforms, job-seeking platforms, epidemic removal processes, and attacked networks.

References:

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