A Unified Framework for Stochastic and Quantum Thermodynamics via Canonical Quantization of Generalized Brownian Motion

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There are two primary frameworks for extending thermodynamics to small systems dominated by fluctuations: stochastic thermodynamics for classical mesoscopic systems and quantum thermodynamics for open quantum systems. In current literature, these two are often treated as independent theories, and the relationship between them remains insufficiently discussed. However, if both frameworks are consistent, quantum thermodynamics should converge to stochastic thermodynamics in the classical limit ($\hbar \to 0$), as is common in bosonic quantum systems. In this study, we propose a systematic method to derive a quantum master equation by applying canonical quantization to a generalized model of Brownian motion specifically constructed to satisfy the laws of stochastic thermodynamics [1].

Our generalized description of the Brownian motion is uniquely defined for any given classical many-body Hamiltonian. Crucially, it is structured to ensure that the first and second laws of stochastic thermodynamics are satisfied regardless of the specific form of the Hamiltonian. The construction of this unified model facilitates a rigorous discussion on the correspondence between fundamental thermodynamic quantities—specifically work and heat—defined in both stochastic and quantum thermodynamics.

We further demonstrate that for quadratic Hamiltonians, the derived quantum master equation reduces to the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation under specific parameter-tuning conditions, ensuring complete positivity [2]. The resulting GKSL dynamics provides a consistent framework for defining work and heat that adheres to the established principles of quantum thermodynamics, ensuring compatibility with the laws of thermodynamics even in the quantum regime. These results provide a new perspective on the quantum-classical correspondence in thermal relaxation processes, extending from many-body systems to scalar field theories [3].

References: [1] T. Koide and F. Nicacio, Phys. Lett. A 494, 129277 (2024). [2] F. Nicacio and T. Koide, Phys. Rev. E 110, 054116 (2024). [3] T. Koide and F. Nicacio, Phys. Rev. E 112, 024127 (2025).