Application of g-subdiffusion equation to model superdiffusion

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The g-subdiffusion equation contains a fractional time Caputo derivative with respect to another function g. This equation can be interpreted as the "ordinary" fractional subdiffusion equation in which the time variable t has been changed to $g(t)$; the function g increases from zero to infinity [1]. By appropriately choosing the function g, the g-subdiffusion equation can be used to describe various types of anomalous diffusion defined by the time evolution of the mean squared displacement (MSD) of a diffusing molecule [2]. Superdiffusion is defined as a random walk of a molecule such that the temporal evolution of the MSD is a power function of time with an exponent greater than one. The probability density of finding a molecule at point x at time t (Green's function) for the g-subdiffusion equation describing superdiffusion coincides in the long-time limit with the Green's function for the "classical" superdiffusion equation with fractional Riesz-Weyl spatial derivative [3,4]. This is surprising because the above-mentioned equations contain fractional derivatives with respect to different variables and therefore have different properties and interpretations. The fractional Riesz-Weyl superdiffusion equation is derived from the CTRW model, in which the probability density of the molecule jump length has a heavy tail. This equation is nonlocal with respect to the spatial variable, which makes it difficult to apply standard local boundary conditions at partially permeable thin membranes. The g-subdiffusion equation describing superdiffusion is local in space; the superdiffusion effect is generated by the rapid increase in the molecule hopping frequency. This equation can therefore be used to describe superdiffusion in membrane systems. The use of the g-subdiffusion equation to model superdiffusion in membrane systems will be demonstrated.

References:

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