Not all chaos is the same

Malgorzata Krawczyk[1]

[1]AGH University, Krakow, Poland

Analysis of the complex dynamics of the system described by fractional-order nonlinear differential equations is not trivial. In this work, we analyze a five-neuron system within the Hopfield model framework. Most of the methods used to study the dynamics of such systems are based on the obtained time series. These methods include the Fourier transform [1], the 0–1 test for chaos [2], and the Birkhoff weighted average [3], which enable identification of periodic, quasi-periodic and chaotic regimes. An important component of nonlinear system analysis is the determination of Lyapunov exponents. In our study, these were calculated from time series using the Sano–Sawada method [4], and the results were compared with those obtained using a perturbation-based approach [5], involving the evolution of both the original and suitably perturbed trajectories. Although these methods provide valuable insight into description of the system's dynamics for specific parameter sets, some aspects remain difficult to capture. Notably, transforming a one-dimensional time series into a two-dimensional representation (as shown in Fig. 1, where $p(t+1) = p(t) + y(t)\cos(ct)$ and $q(t+1) = q(t) + y(t)\sin(ct)$), performed in the first step of the aforementioned 0–1 test for chaos, reveals differences in system behavior that are otherwise not apparent. In particular, it shows that simply classifying dynamics as chaotic is insufficient, as different realizations of this regime may vary significantly.

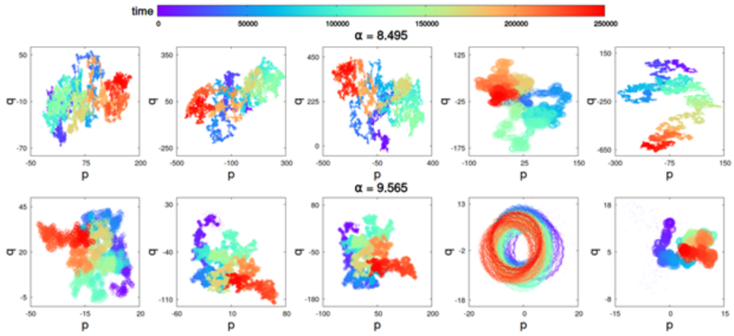

Fig. 1 Time evolution of neurons y_i in the (p,q) space for two values of the control parameter α

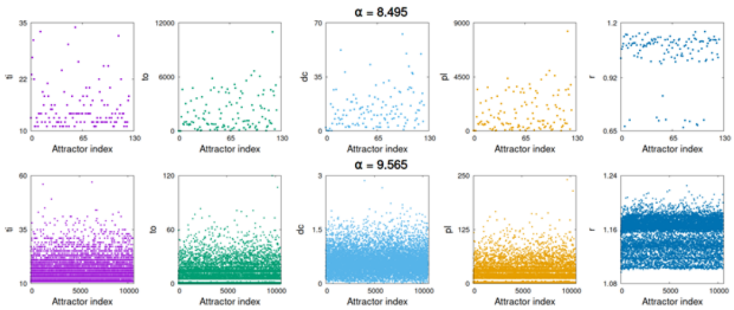

Fig. 2 Measures of the system in the (p,q) space for a single neuron: time spent inside the attractor (ti) and its radius (r); time spent outside the attractors (to); and the path length (pl) and Euclidean distance (dc) between successive attractors.

To address this limitation, we propose a complementary approach that enables a more detailed characterization of the system. The observed dynamics exhibit the presence of local attractors in (p, q) space. To characterize both the local properties of individual attractors and the transitions between them, the following measures of the system for a single neuron are considered: the time spent inside the attractor and its radius, the time spent outside the attractors, and the path length and Euclidean distance between successive attractors. The observed random walk in the (p, q) space for two different values of control parameter α, illustrated in Fig. 1, provides a detailed characterization of the chaotic states in the fractional-order Hopfield network. The dynamics reveal two distinct cases of chaotic neuronal behavior, manifested as two types of random walks with significantly different values of proposed measures, as presented in Fig. 2.

References:

[1] J.B. Joseph Fourier, The Analytical Theory of Heat, Nature 18, 192 (1878)

[2] G.A. Gottwald and I. Melbourne, On the Implementation of the 0–1 Test for Chaos, SIAM Journal on Applied Dynamical Systems 8(1), 129 (2009)

[3] S. Das et al., Measuring quasiperiodicity, EPL 114, 40005 (2016)

[4] M. Sano and Y. Sawada, Measurement of the Lyapunov Spectrum from a Chaotic Time Series, Physical Review Letters 55(10), 1082 (1985)

[5] D.C. Soriano et al., A method for Lyapunov spectrum estimation using cloned dynamics and its application to the discontinuously-excited FitzHugh–Nagumo model, Nonlinear Dynamics 67, 413 (2012)