

# Fluctuations of information flow: exactly solvable cases

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We present a trajectory-level framework for information thermodynamics in autonomous nonequilibrium systems by formulating stochastic information flow (SIF) as a fluctuating observable in bipartite Langevin dynamics. In this setting, average information flow modifies the apparent second law for subsystems, so that one component can exhibit negative apparent entropy production while the full system remains thermodynamically consistent. We extend this idea beyond average behavior and ask how rare information-flow fluctuations organize the dynamics of autonomous engines at the level of individual trajectories. To answer this, we combine stochastic thermodynamics with large-deviation theory and develop an efficient computational approach for linear Langevin systems, including underdamped dynamics with inertia.

We first illustrate the basic physics in a minimal model of two elastically coupled particles in contact with reservoirs at different temperatures. In this example, the mean and variance of SIF both grow with coupling strength, showing that SIF is a sensitive probe of interaction between subsystems. At the same time, inertia suppresses information exchange: increasing mass causes both the mean SIF and its fluctuations to saturate earlier and at smaller values. This identifies inertia as a physical mechanism that limits how effectively one subsystem can learn about and respond to the other, while also revealing multiple timescales in finite-time information-flow fluctuations.

We then apply the framework to linear active Brownian engines and derive a modified information-flow arbitrage relation that bounds the extracted work by the active input work and the average SIF. This gives a transparent subsystem interpretation of engine operation: one subsystem functions as an information-driven engine or energy transducer, while the other acts as an autonomous information processor that measures the first subsystem and feeds back on it. In this way, information flow becomes not only a correction to subsystem thermodynamics but also a quantitative descriptor of internal engine architecture.

A central result is summarized in the figure, where we analyze the large deviations of SIF in an overdamped active engine through the rate function, scaled cumulant generating function, work-information relation under tilted ensembles, and the associated biased probability currents. We show that trajectories with strongly negative SIF are precisely those associated with enhanced work extraction, whereas biasing toward positive SIF reverses the probability-current structure and turns the engine into a work-absorbing device. The same figure also identifies a dynamical phase transition in the fluctuation statistics, signaling an abrupt change in the dominant trajectory structure responsible for rare events. Extending the analysis to the underdamped regime, we further show that increasing inertia weakens this dynamical phase transition and can eliminate it altogether. These results establish SIF as a practical observable for diagnosing how autonomous microscopic engines convert information exchange into useful work on a trajectory-by-trajectory basis.

