Bulk-boundary decomposition of neural networks

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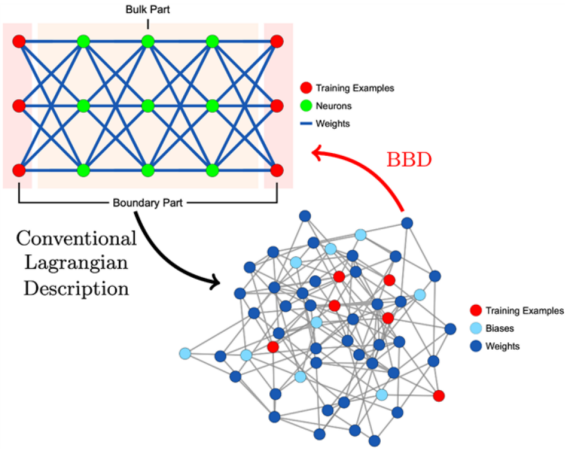
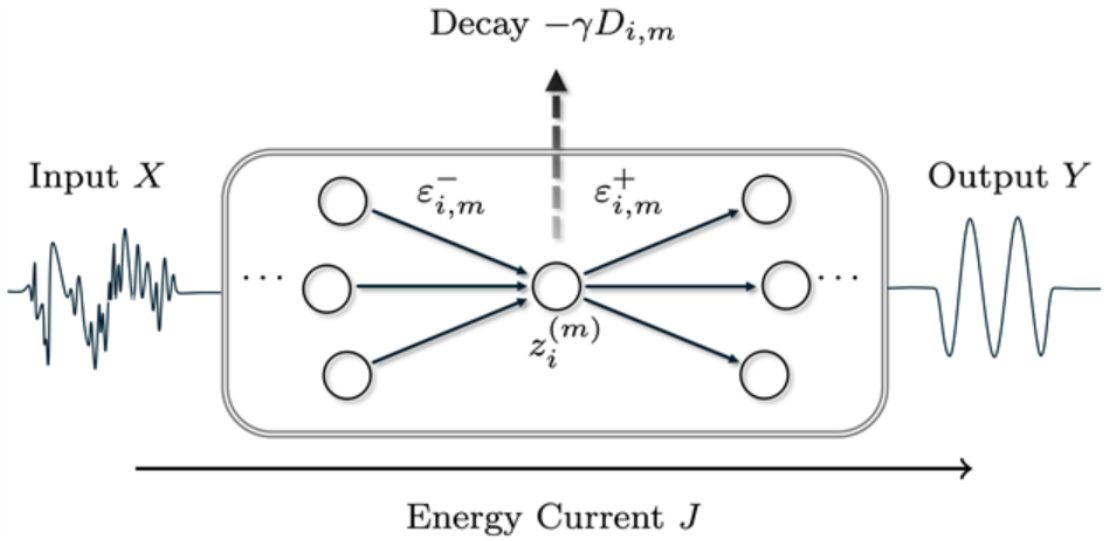


We introduce the bulk-boundary decomposition as a framework for studying the training dynamics of deep neural networks. Starting from the stochastic gradient descent formulation, we show that the Lagrangian can be separated into two parts: a bulk term that does not depend on data and a boundary term that depends on training samples. The bulk describes the intrinsic dynamics determined by the network architecture and activation functions, while the boundary represents the stochastic effects introduced by data at the input and output layers. This decomposition reveals that deep neural networks possess a local and homogeneous structure. As a consequence of this structure, we derive an energy continuity equation inside the network. Also, the local and homogeneous structure enables the application of statistical physics techniques, allowing macroscopic analyses to be carried out.



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