A solvable model for a generic aggregation process: Theoretical utopia or emerging reality?

Michał Łepek[1]

[1]Warsaw University of Technology, Warsaw, Poland

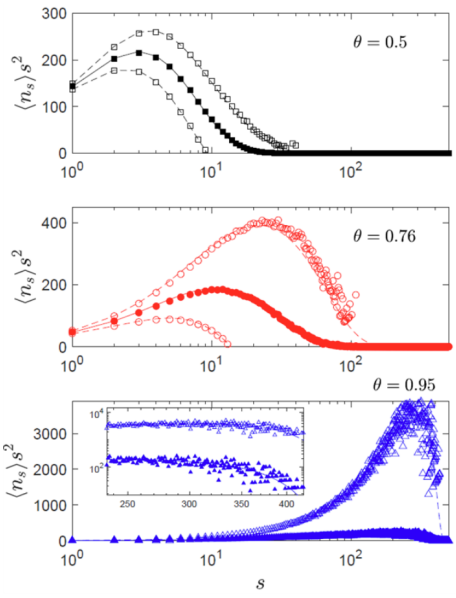
Aggregation processes appear across physics, chemistry, planetary science, and biology, yet constructing a unified, predictive theory for coagulating systems remains notoriously difficult. Traditional frameworks — the Smoluchowski kinetic equations and the stochastic Marcus–Lushnikov model — provide powerful but incomplete perspectives. The former relies on assumptions such as infinite system size, and continuous concentrations, while the latter captures finite-size features and stochasticity but becomes analytically intractable beyond the simplest kernels. In recent years, however, a combinatorial route to irreversible aggregation has emerged as a promising alternative [1], enabling exact or approximate analytical predictions for a broad spectrum of coagulation kernels.

This approach replaces differential-equation dynamics with the direct counting of all microstates leading to a given system configuration. It hinges on three ingredients: (i) enumerating partitions of monomers into clusters, (ii) computing all possible growth histories for each cluster, and (iii) counting the admissible interleavings of coagulation acts in discrete time. These elements culminate in closed-form expressions for state probabilities and yield not only the average number of clusters of a given size but also their variance and even full cluster-number distributions — quantities inaccessible to classical theories.

Despite its limitations [2], the combinatorial method reproduces exact results for the constant and additive kernels and yields accurate approximations for several physically relevant cases, including the linear-chain kernel, and Brownian aggregation kernels. Even when finding explicit analytical forms fail, the recursive formulation produces predictive finite-size distributions for any kernel $K(i, j)$. This enables tractable modelling of rich processes ranging from planetesimal growth and aerosol coagulation to polymerization, protein aggregation, and field-induced chaining in electrorheological and magnetorheological fluids.

A central and still underappreciated aspect — and one we will highlight in the presentation — is how combinatorial and, more generally, Marcus–Lushnikov solutions can be explicitly related to Smoluchowski solutions formulated in physical time. By exploiting the transformation between discrete aggregation steps and the continuous Smoluchowski timescale, we will show how to meaningfully compare predictions across these frameworks, even though their foundational assumptions differ strongly. This mapping is not broadly known in the community, yet it is essential for interpreting finite-system theories in experimentally relevant units.

Altogether, these developments suggest that a universal, analytically tractable model of aggregation — long viewed as a theoretical utopia — may approach reality (at least, to some extent).



Protoplanetary aggregation. Average number of clusters of given size, multiplied by its squared size, and corresponding standard deviation for the gravitational aggregation kernel [3] for the three stages of the aggregation process.

References:

[1] A. Fronczak, A. Chmiel, P. Fronczak: Phys. Rev. E 97, 022126 (2018), Exact combinatorial approach to finite coagulating systems. https://doi.org/10.1103/PhysRevE.97.022126

[2] F. Leyvraz: Phys. Rev. E 106, 024133 (2022), Rate equation limit for a combinatorial solution of a stochastic aggregation model. https://doi.org/10.1103/PhysRevE.106.024133

[3] M. Łepek, A. Fronczak, P. Fronczak: Physica D 489, 135097 (2025), A mini-review on combinatorial solutions to the Marcus–Lushnikov irreversible aggregation. https://doi.org/10.1016/j.physd.2025.135097