A fast and flexible algorithm to measure fractality in complex networks

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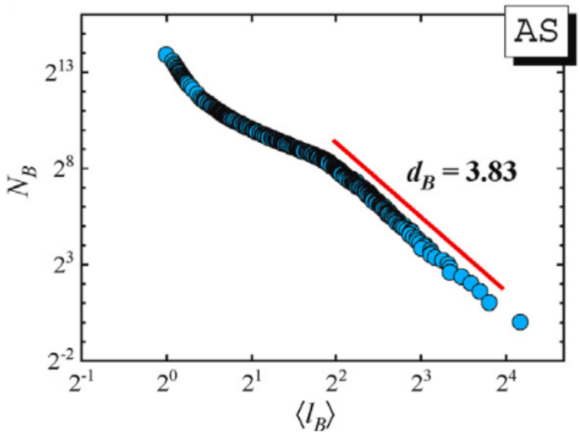
Quantifying the fractal structure of complex networks typically relies on box covering methods that fix the box diameter and then count the minimum number of boxes required to cover the system [1]. While foundational, this paradigm inherits limitations: box sizes must be integers, box size distributions are susceptible to yield unpredicted characteristics, and computational cost grows rapidly with system size. Recently, we introduce a conceptually inverted and computationally efficient alternative — the Fixed Number of Boxes (FNB) algorithm — which determines fractal dimensions by fixing the number of boxes first while allowing their diameters to adjust flexibly [2]. This methodological reversal produces smoother scaling curves, dramatically increases resolution, and reveals fractality in systems previously inaccessible to classical algorithms.

The FNB method is theoretically grounded in the recently proposed generalized scaling theory of fractal complex networks [3], where geometric self similarity depends jointly on spatial scale and local structural heterogeneity. Motivated by the role of hub hub repulsion, latent geometric embeddings, and hidden metric spaces, our algorithm assigns nodes to boxes through graph Voronoi diagrams centered on local hubs. This approach naturally aligns with the hypothesis that network fractality emerges from an underlying geometric organization: high degree nodes act as attractors for local structure, while Voronoi cells approximate metric basins in hidden space.



Determining the fractal dimension of the Internet network (AS-level) using the FNB algorithm (number of boxes versus the average box diameter) [2]

We test the algorithm across nine networks — including model generated fractal graphs [4], real world systems with confirmed fractality, networks whose fractal status was previously ambiguous (e.g., the Internet at the autonomous systems, AS, level, and human protein–protein interaction network), and even extremely large systems beyond the reach of traditional greedy coloring (up to 2.1 M nodes). In all cases, the FNB algorithm produces stable scaling functions, homogeneous box size distributions, and significantly improved computational performance (scaling as O(N) per data point rather than O(N^2). For networks where theory predicts the mass and degree exponents of fractal boxes, FNB coverings satisfy the microscopic scaling relations implied by generalized self similarity, providing non trivial theoretical validation.

The FNB algorithm uses continuous mean distances rather than integer diameters, guarantees connected boxes via BFS "burning," and generalizes naturally to weighted or directed networks. The method also suggests deeper connections between fractality, modularity, and network geometry, hinting at a unified framework linking fractal boxes, Voronoi partitions, and hierarchical community structure.

References:

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