Thermal equilibrium with generalized time-reversal symmetry

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Determining whether a stochastic system is at equilibrium requires a careful characterization of its behavior under time reversal. In particular, identifying the correct transformation properties of dynamical variables is essential to assess detailed balance and to define the entropy production rate in a consistent and unambiguous way. In this talk, I will discuss stochastic processes whose underlying time-reversal symmetry cannot be reduced to the standard parity prescription, namely the simple inversion of momentum variables. I will present a systematic framework to construct equilibrium Langevin dynamics starting from their reversible deterministic counterparts, ensuring consistency with the appropriate notion of time-reversal symmetry. Building on this approach, I will show that for a class of systems characterized by multiple timescales the slow degrees of freedom evolve, in a suitable limit, according to a Langevin equation satisfying the generalized form of detailed balance. This result provides a direct link between microscopic reversibility and equilibrium behavior at coarse-grained scales. Moreover, I will discuss an unambiguous criterion to assess whether a given stochastic dynamics is genuinely at thermal equilibrium. The method applies to a broad class of stochastic processes evolving through Langevin equations, encompassing in particular all two-dimensional systems and, more generally, systems admitting an integrable deterministic limit. In these cases, the nontrivial time-reversal symmetry is naturally identified in action-angle variables. As an illustrative example, I will consider the Lotka-Volterra model. Starting from a reversible many-body deterministic population dynamics, I will show that the underlying time-reversal symmetry constrains the symmetry properties of the correlation functions of observables of the slow variables. Through the proposed criterion, this symmetry allows one to construct a stochastic formulation consistent with equilibrium thermodynamics. In contrast, I will show that commonly used stochastic versions of the same model break time-reversal symmetry and are therefore intrinsically out of equilibrium, as evidenced by a non-zero entropy production rate. Overall, these results highlight the role of time-reversal symmetry both in deriving effective equilibrium dynamics and in assessing the consistency of stochastic models in general settings.

Reference: Lucente, D., Baldovin, M., Viale, M., Vulpiani, A. (2025). Langevin dynamics with generalized time-reversal symmetry. arXiv preprint arXiv:2504.05980.