

# Are we entering the "Houdayer era" for spin glass simulations?

**Victor Martin-Mayor**<sup>1</sup>

<sup>1</sup>Universidad Complutense De Madrid, Madrid, Spain

Finding the minimum of a spin glass Hamiltonian is an NP-complete problem, but sampling from the Boltzmann distribution at low temperatures seems not to be any easier. Indeed, while ground states were computed on simple-cubic lattices with linear size  $L=14$  already in 2001 [1], only in 2026 has it been possible to equilibrate systems of similar or larger sizes at temperature  $T=0.2 \approx 0.21T_c$  (see the talk by Claudio Chilin and Ref. [2]). The technical step forward that makes it possible to equilibrate at such a low temperature is Houdayer's cluster algorithm [3].

Houdayer's cluster move provides a finite-temperature generalization of a genetic algorithm. It was thought for many years to be useful only for simulations of the square lattice, although Ref. [4] suggested that also simulations in space dimension  $D=3$  could benefit from it. Indeed, we have recently found that the cluster move produces a significant speed-up in a cubic geometry [2], and an orders-of-magnitude dynamical acceleration on a rectangular right-prism geometry (see talk by Isidoro González-Adalid and Ref. [5]).

The talk will discuss possible mechanisms underlying this significant performance difference, with an emphasis in the geometry of the clusters. Some important tricks that boost performance when the cluster's geometry is unfavourable [2] will be described. Time permitting, we shall describe as well some recent numerical experiments exploring the effectiveness of the cluster moves in other contexts (particularly space dimensions other than  $D=3$  and off-equilibrium simulations).

## References:

[1] E. Marinari and G. Parisi, Effects of a bulk perturbation on the ground state of 3d Ising spin glasses, *Phys. Rev. Lett.* 86, 3887 (2001).

[2] C. Chilin, E. Marinari, V. Martin-Mayor, G. Parisi, J.J. Ruiz-Lorenzo and D. Yllanes, manuscript in preparation (2026).

[3] J. Houdayer, A cluster Monte Carlo algorithm for 2-dimensional spin glasses, *The European Physical Journal B - Condensed Matter and Complex Systems* 22, 479 (2001).

[4] Z. Zhu, A. J. Ochoa, and H. G. Katzgraber, Efficient cluster algorithm for spin glasses in any space dimension, *Phys. Rev. Lett.* 115, 077201 (2015).

[5] M. Bernaschi, L. A. Fernández, I. González-Adalid Pemartín, V. Martín-Mayor, G. Parisi, and F. Ricci-Tersenghi, Low energy excitations in a long prism geometry: computing the lower critical dimension of the Ising spin glass, arXiv:2601.07926.