Information-Geometric Structures Induced by Generalized Relative Entropies

Hiroshi Matsuzoe[1]

[1]Nagoya Institute Of Technology, Nagoya, Japan

Information geometry studies spaces of probability measures, or statistical models, as differentiable manifolds and investigates the natural mathematical structures arising from this geometrical viewpoint. Nowadays, it has developed into an interdisciplinary field with broad applications in statistics, information theory, and machine learning. Typical geometric objects in this framework include Riemannian metrics, affine connections, and divergence functions, which provide geometric interpretations of fundamental concepts such as estimation, hypothesis testing, and learning.

Information-geometric structures are commonly constructed based on two fundamental principles: invariance under transformations of the sample space and dual flatness of a pair of affine connections. Invariance ensures that the resulting geometric description is independent of arbitrary choices of coordinates or representations, thereby providing a consistent theoretical foundation. Dual flatness, characterized by the existence of mutually dual affine connections and associated potential functions, plays a crucial role in simplifying computations and enabling efficient methods for statistical inference and optimization.

Divergence functions serve as generalized distance measures in information geometry. Under appropriate regularity conditions, their second-order derivatives induce a Riemannian metric, typically the Fisher information metric, while higher-order derivatives determine a pair of dual affine connections. A canonical example is the Kullback–Leibler (KL) divergence, or relative entropy, which induces the Fisher information metric together with invariant, dually flat affine connections on exponential families. Replacing the divergence function leads to distinct geometric structures, motivating extensive research on alternative divergences.

In particular, generalized relative entropies arising in generalized statistical mechanics, such as the Tsallis entropy, have been extensively studied in connection with non-equilibrium and complex systems. The α-divergence in information geometry and the Tsallis relative entropy are closely related and, under suitable parameter correspondences, give rise to essentially equivalent geometric structures. However, important differences arise in their regularity properties. When the framework is extended to non-regular statistical models, these differences become significant, and the geometry induced by the Tsallis relative entropy often provides a more natural formulation.

In this talk, we revisit information-geometric structures induced by generalized relative entropies and investigate their extensions to broader classes of statistical models, including non-regular cases.