Hyperscaling relations of spatial fluctuations in urban populations

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The science of urban scaling has uncovered many striking regularities in cities, ranging from the scaling of socio-economic activity with population to fractal patterns in urban morphology. Yet most work has focused on how aggregate urban indicators scale with total population across cities, and much less attention has been given to the scaling of spatial fluctuations within cities. Understanding these fluctuations is important for linking urban morphology to the spatial organization of population and activity.

In this work we examine cities as spatially extended complex systems and study how population fluctuations scale across distances. Using high-resolution gridded population data, we apply a coarse-graining procedure analogous to a phenomenological renormalization group: the population field is recursively aggregated into square blocks of increasing size, and we measure how the mean and variance of the population per block scale with the aggregation length scale. This analysis yields two scaling exponents describing the multiscale structure of urban populations: an exponent governing the scaling of the mean population with block size, which in the small-scale limit approaches the planar fractal dimension of urban occupancy, and an exponent governing the scaling of population fluctuations.

Across urban regions in the Netherlands, we find that these exponents are not independent. Instead, cities cluster close to a linear hyperscaling relation in the (,) plane. Moreover, the position of cities along this line varies systematically with their degree of urban development, indicating that multiscale population structure encodes information about urban density and spatial organization. To test the generality of this finding, we extend the analysis to several hundred cities worldwide using the Global Human Settlement population dataset spanning multiple continents and decades. The hyperscaling relation persists across this much broader sample, although its slope and intercept vary across geographic regions and drift systematically over time.

To interpret these observations, we develop a theoretical framework based on the decomposition of the coarse-grained variance into independent and covariance-driven contributions. A mean-field approximation with independent cells predicts a quadratic variance–mean relation and fails to reproduce the observed exponent interdependence, indicating that spatial correlations play a central role. Incorporating long-range spatial correlations in the population field leads to a scaling form in which the fluctuation exponent is controlled by a correlation dimension. In the correlation-dominated regime, the theory predicts $\gamma = 2 + D_c$. If mature urban systems approach monofractal organization such that the correlation dimension coincides with the planar fractal dimension ($D_c \simeq \beta$), an asymptotic hyperscaling relation $\gamma \simeq 2 + \beta$ emerges. Empirically, we observe a temporal drift of the hyperscaling relation toward this limit as cities grow and develop. The resulting interdependence between exponents links urban morphology and spatial fluctuations, providing new constraints for mechanistic models of urban growth and for the scaling behavior of spatial indicators derived from local population statistics.