Dual thermodynamic uncertainty relations: dissipation-coherence trade-off and thermodynamic speed limit for noisy limit cycles

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The development of stochastic thermodynamics allows us to clarify the relationship between a system's function and its associated energetic dissipation, measured with the entropy production (EP). Two notable examples are thermodynamic uncertainty relations (TURs) and thermodynamic speed limits (TSLs). The TURs establish the trade-off relation between the EP and the precision of currents, while the TSLs provide another trade-off relation between the EP and the speed of the transition. These two types of trade-offs are closely related: We can derive the TSLs and TURs from certain common inequalities. We can also obtain the TSL from the short-time limit of the TUR (so called the short-time TUR [1]).

Another function attracting attention is the sustainability of the phase correlation (coherence) in noisy oscillations. Maintaining coherence requires thermodynamic cost. This fact is described by the dissipation-coherence trade-off (DCT), which was originally proposed for Markov jump processes [2]. The DCT states that the entropy production required for one oscillatory period is bounded by the number of oscillations that occur before steady-state correlations are disrupted. Despite significant theoretical developments in this area, this DCT remains unproven. The relationship between the DCT and other trade-offs, such as TURs and TSLs, is also unclear.

Recently, Santolin and Falasco discovered the DCT for the stochastic limit cycles in the weak-noise regime [3]. They analytically derived the DCT for the systems satisfying one of the following two conditions: (i) the diffusion coefficient matrix is proportional to the identity matrix, or (ii) the system is near the critical point of the Hopf bifurcation. For systems without these conditions, an analytic proof is still lacking, although numerical validation has been performed.

In this talk, we derive the DCT for the stochastic limit cycles in the weak-noise regime, independent of the conditions imposed in the proof by Santolin and Falasco [4]. The DCT is obtained by substituting an observable into the short-time TUR [1]. We also derive a new TSL for the stochastic limit cycles by substituting another observable, which is the dual of the one for the DCT, into the TUR. This implies that, in the weak-noise regime, the DCT is the dual of the TSL in the sense of observable used in the TUR. Additionally, we introduce the connection between the DCT and phase reduction theory. Using the asymptotic phase, we construct the diffusion coefficient matrix that saturates the DCT.

References:

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