Criticality and Temporal Complexity in a Stochastic Exponential Dense Associative Memory Model

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Associative memory models represent one of the earliest paradigms of learning algorithms, enabling the retrieval of stored patterns from corrupted inputs. Dense Associative Memory (DAM) models incorporating higher-order interactions have been proposed to dramatically extend storage capacity and robustness. More recently, exponential interaction functions have further enhanced these capabilities, giving rise to modern DAM architectures whose storing capacity and equilibrium critical properties are still under active investigation. However, the out-of-equilibrium temporal dynamics emerging during the retrieval process under stochastic perturbations remain largely unexplored.

In this work, we investigate a stochastic exponential Dense Associative Memory (SEDAM) model with multiplicative salt-and-pepper noise, explicitly modeling retrieval as a dynamical process evolving in time. We analyze how noise intensity and memory load jointly shape the network's behavior, focusing on both classical order parameters and temporal self-organization. Using both the average overlap parameter, which is a standard measure of retrieval performance, and diffusion scaling indices H and δ as order parameters, we identify critical regimes separating retrieval and non-retrieval dynamics. The parameter H, which is a proxy for temporal correlations, is evaluated through the Detrended Fluctuation Analysis (DFA). DFA is applied to the temporal evolution of the overlap parameter. Numerical simulations show that the critical noise threshold decreases with increasing memory load, while the critical regime is characterized by long-range temporal correlations and persistent memory effects. When trained on structured datasets such as MNIST, the SEDAM exhibits highly correlated temporal dynamics near criticality, with $H \simeq 1.3$, indicating strong temporal persistence. Comparisons with a stochastic version of the classical Hopfield network and with DAMs trained on random Rademacher patterns show qualitatively similar behaviors but over broader noise ranges and with larger diffusion exponents ($H \simeq 1.5$).

We further analyze MNIST-based SEDAM dynamics through the lens of Temporal Complexity (TC), a framework capturing intermittent transitions between order and disorder and scale-free temporal statistics. Transition events associated with the birth and death of neural avalanche-like structures are extracted and compared with coincidence-based events. TC indicators are the scaling H and the self-similarity index δ of the diffusion distribution, this last one evaluated through the Diffusion Entropy (DE) method. TC indicators reveal that SEDAM displays regimes of self-organized intermittency emerging over finite intervals of noise intensity rather than at isolated critical points, consistent with the concept of extended criticality. These regimes become accessible at slightly lower noise levels as memory load increases, linking storage capacity to the network's ability to self-organize dynamically.

Overall, our results suggest that stochastic exponential DAMs display interesting dynamical critical behavior, where retrieval performance, temporal correlations, and self-organization appear to be closely related, offering some insights into learning and information processing in artificial and biological neural systems.

References:

[1] Cafiso, C. and Paradisi, P. (2026). Criticality of a stochastic modern Hopfield network model with exponential interaction function. Submitted to Physical Review E. ArXiv preprint https://arxiv.org/abs/2509.17152.

[2] Cafiso, C. and Paradisi, P. (2026). Temporal complexity and self-organization in an exponential dense associative memory model. Submitted to Complex and Intelligent Systems. ArXiv preprint https://arxiv.org/abs/2601.11478.

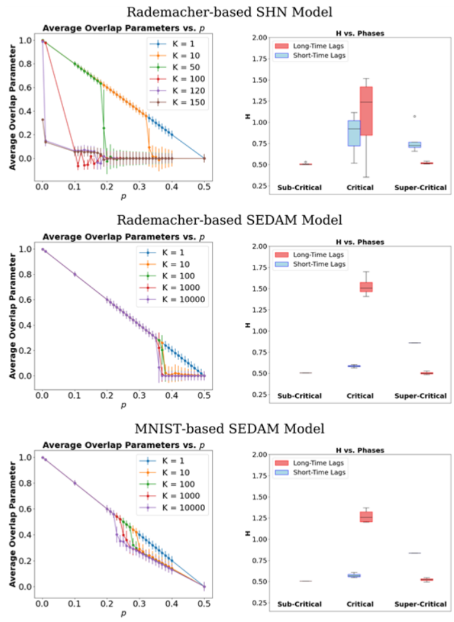

Fig. 1: Criticality Results. Average overlap parameter against p by increasing the number K of stored patterns (on the left), and statistical boxplots of second moment scaling H estimated from DFA in the three different phases (sub-critical, critical, super-critical) for both short and long time lag regimes (on the right) for all the model studied: Rademacker-based Stochastic Hopfield Network Model (SHN) (Top Panels), Rademacker-based Stochastic Dense Associative Memory Model (SEDAM) (Middle Panels), and MNIST-based SEDAM (Bottom Panels).

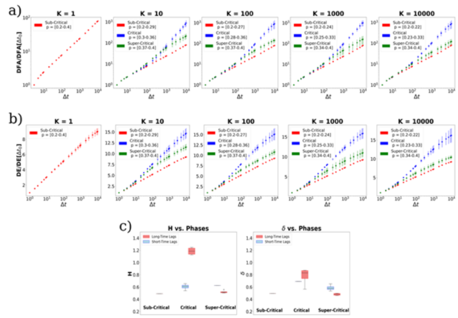

Fig. 2: Avalanches events. (a) DFA curves in the three different phases (sub-critical, critical, super-critical) for $K = 1, 10, 100, 1000$ stored patterns. (b) DE curves in the three different phases (sub-critical, critical, super-critical) for $K = 1, 10, 100, 1000$ stored patterns. Each panel legend shows the range of p values relating to the phase indicated in the legend itself. DFA and DE are referenced to the values $DFA(\Delta t_0)$ and $DE(\Delta t_0)$, respectively, being Δt_0 the smallest time lag at which DFA and DE are computed. The reported DFA and DE functions are derived as averages (points) and standard deviations (vertical bars) computed over the range of noise intensity values p reported in the figure legend. (c) Boxplots of H (left) and δ (right) values in the three phases (sub-critical, critical, super-critical) for both short- and long-time lag regimes. The crossover between short- and long-time ranges decreases with K.